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A BRIEF SURVEY ON FUZZY SET INTERPOLATION METHODS

***Zsolt Csaba Johanyák*[1], *Szilveszter Kovács*[2]**

[1]Senior lecturer, [2]PhD, Associate professor

[1]*Kecskemét College, Mechanical Engineering and Automation College Faculty,*
Department of Information Technology,

[2]*University of Miskolc, Faculty of Mechanical Engineering,*
Department of Information Technology

Fuzzy Set Interpolation (FSI) methods aim the determination of a new linguistic term in a given position of a fuzzy partition. They are applied mostly for the calculation of the antecedent and consequent sets of the new rule in the so-called two-step Fuzzy Rule Interpolation (FRI) based reasoning methods. This paper surveys three FSI techniques outlining their essential features.

1. INTRODUCTION

Fuzzy systems working with classical compositional rule based reasoning methods require the existence of applicable rules for all the possible observations, i.e. an $\varepsilon>0$ covering rule base. In several cases this demand cannot be fulfilled either due to lack of proper rules or the limited storing and processing possibilities. In such circumstances the application of FRI based inference techniques offer the best solution.

FRI methods can be divided into two groups depending on whether they produce the conclusion directly or a new auxiliary rule is interpolated first. The members of the second group, the so-called two-step methods essentially follow the concepts of the Generalized Methodology of fuzzy rule interpolation (GM) introduced by Baranyi et al in [1].

The rest of this paper is organized as follows. Section 2 recalls the basic ideas of GM. A condition set for the evaluation and comparison of the FSI techniques is introduced in section 3. Sections 4-6 present the FSI methods FEAT-p, FEAT-LS and VESI.

2. GENERALIZED METHODOLOGY

The generalized methodology of fuzzy rule interpolation (GM) [1] was introduced by Baranyi et al. Its basic idea is that it divides the task of rule interpolation into two steps. First a new rule is interpolated, of which antecedent part contains linguistic terms, which overlap the fuzzy sets of the observation at least partially in each antecedent dimension and the reference point of each antecedent linguistic term - considering a multidimensional antecedent universe of discourse - has the same abscissa as the respective set of the observation.

Hereunder the reference point (RP) is used form the characterization of the position of the linguistic terms. Most often applied choices for its selection are the centre of the core (RP_{CC}) [1][2][5], the centre of the support (RP_{SC}) [2], the centre of gravity (RP_{GC}) [3] and the unweighted or weighted average of the abscissas of the characteristic (break) points of the shape (RP_{UAV}, RP_{WAV}) [3]. Figure 1 presents the listed reference point types in case of a trapezoid shaped fuzzy set. The distance of the fuzzy sets is measured as a Euclidean distance between their reference points.

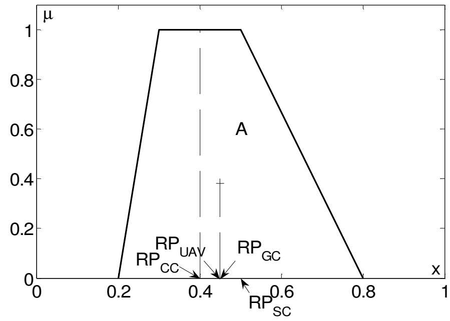

Figure 1.
Possible reference points in case of a trapezoidal shaped linguistic term

The new rule is determined in three stages. First the antecedent part is calculated using a set interpolation method. Next the position of the consequent sets is determined e.g. based on the fundamental equation of the rule interpolation (FERI) [10]. Thirdly the shape of the consequent linguistic terms is calculated using the same technique as in the first stage.

In the second step of the GM the conclusion of the inference process is determined by firing the new rule. Usually the antecedent part does not fit perfectly the observation in each dimension and therefore some kinds of special single rule reasoning techniques are needed in order to obtain the desired result.

3. CONDITIONS ON FUZZY SET INTERPOLATION METHODS

In order to facilitate the evaluation and comparison of the techniques being surveyed we have compiled a set of conditions based on the *General conditions on rule interpolation methods* introduced in [4]. They are the followings.

1. *Avoidance of the abnormal set shape.* The estimated fuzzy set should be a valid one.
2. *Compatibility with the partition* (interpolation capability). If the point of the interpolation coincides with the reference point of one of the linguistic terms of the partition the resulting set should be also identical with that fuzzy set.
3. *Conserving the piece-wise linearity*. If the sets of the partition are pice-wise linear with break-points situated at the same α-level the interpolated set should conserve this feature.
4. *Applicability without any constraint regarding to the shape of the fuzzy sets*. This condition can be lightened practically to the case of polygons, since piece-wise linear sets are most frequently encountered in the applications.
5. *Extrapolation capability.* The method should be also able to generate a new set when the interpolation point is outside the range defined by the leftmost and rightmost linguistic terms of the partition.

4. FEAT-p

A common ground of all FSI methods is the assumption that there is regularity between the linguistic terms of a partition. The Fuzzy sEt interpolAtion Technique based on polar cuts [7] goes out from the assumption that a better set approximation can be attained by taking into consideration not only the two sets flanking the observation/conclusion but all the available linguistic terms in the partition. Besides, this approach makes possible the technique FEAT-p to be applied for extrapolation, too.

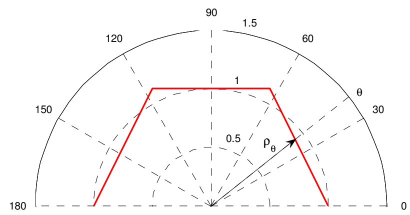

Figure 2
Polar cut

The method uses the centre of the core as reference point. First all sets are shifted virtually horizontally in order to reach the coincidence of their reference points with the position of the interpolation (see fig. 3). The term virtual expresses that the linguistic terms do not change their position permanent. Their shifted version is used only for the calculation of the interpolated set. Next the shape of the new set is determined from the collection of the overlapped sets by applying the extension and resolution principle of fuzzy sets asserted to the case of polar cuts.
A polar cut (fig. 2) is defined by a value pair $\{\rho, \theta\}$ that determines a point on the shape of the linguistic term. The value ρ denotes the polar distance at the angle θ. The polar distances defining the shape of the interpolated linguistic term are calculated as weighted averages of the polar distances of the shifted sets for the same θ angle.
The weighting expresses that the sets whose original position were in the neighbourhood of the interpolation point should exercise higher influence in the calculations as those ones situated in farther regions of the partition.
The advantages of FEAT-p are its applicability for extrapolation as well, is ability to handle cases when one or more sets of the partition are subnormal or non-convex and its comprehensibility. The drawback of the method is that due to the non-conservation of the piece-wise linearity usually it requires a relative high number of polar levels to be calculated. This feature increases the computational need of the method. It ensures the fulfilment of the conditions 1,2,4 and 5.

5. FEAT-LS

In several applications all linguistic terms of a partition belong to the same shape type and the characteristic (break) points are also situated at the same α-levels. In such cases it seems to be a natural condition on the new linguistic term to suit this regularity. Furthermore if one or more shape-pieces delimited by the characteristic points are linear it could be expected that the corresponding pieces of the new set to be linear as well.
The Fuzzy sEt interpolAtion Technique based on the method of Least Squares (FEAT-LS)[6] was developed especially for these purposes. It is applicable for both of the problems interpolation and extrapolation.
Similar to the method presented in the previous section FEAT-LS takes into consideration all the sets of the partition by shifting them into the position of the interpolation (fig. 3). Having the ordinate values predetermined as a characteristic feature of the partition the calculations deal only with the abscissas of the characteristic points of the new linguistic term.

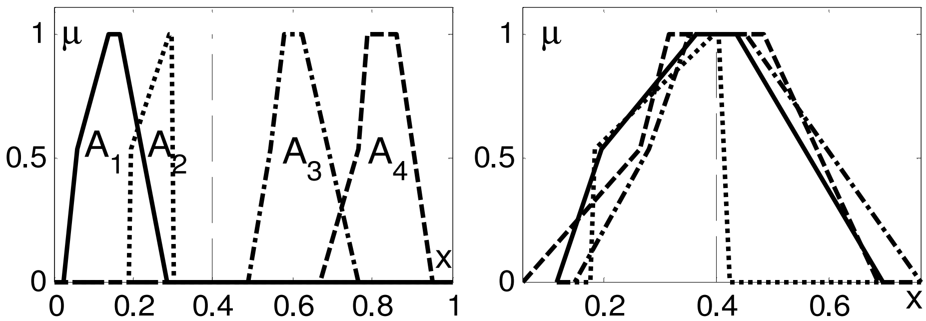

Figure 3
Original partition and the effect of the shifting

They are determined from the corresponding points of the overlapped sets applying the method of weighted least squares in light of the requirement that the resulting set should be a valid one and should fit the typical shape type of the partition. The weighting also here expresses that the neighbouring sets should have a higher influence by the calculations.
The main advantage of the method is its low computational complexity and shape conserving capability. Its drawback is that its application is restricted to the partitions built from uniform shaped linguistic terms. FEAT-LS fulfils the conditions 1,2,3 and 5.

6. VESI

Vague Environment based Set Interpolation [8] applies a significantly different approach for the determination of the shape of the new set. It uses the concept of Vague Environment (VE), which was originally introduced by Klawonn in [9] and first applied for FRI purposes by the one-step method FIVE [11].
The first stage of VESI is the generation of the VE of the partition by (1) calculating a universal scaling function that characterizes the shape of all linguistic terms of the

partition by only one function and (2) defining a set of representative (reference) points holding information about the position of the sets.
Exact scaling functions can be determined only in some regular cases (e.g. Ruspini partition see. fig. 4). Therefore generally approximate scaling functions are applied for this task. The best results have been obtained by the non-linear interpolating scaling function proposed by Kovács in [11].
The interpolation and extrapolation capability of the VE is given by the feature that conform to the extended concept of the VE [11] each interval of the scaling function delimited by prototypical points and the lower and upper endpoints of the range of the partition – including here also the sparse portions of the partition – is defined only by the neighbouring flanks of the sets.

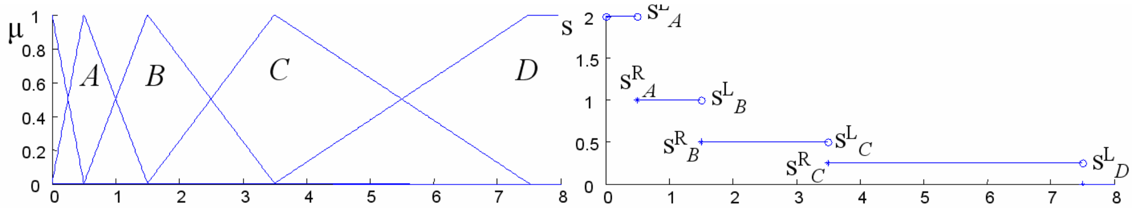

Figure 4
Ruspini partition and its scaling function

The second stage of VESI starts with the given interpolation point assuming that the new set conserves the properties of the VE. Thus one can generate the shape of the new linguistic term from the scaling function easily by handling the interpolation point as a prototypical point.

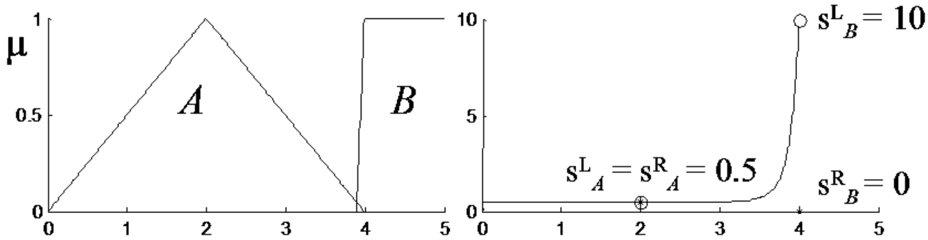

Figure 5
Partition with different set shapes and its VE using non-linear interpolated scaling function

The main advantage of VESI is that it has the lowest computational complexity from the three surveyed methods because the preparatory calculations (stage 1) have to be done only once. As a drawback it can be mentioned that due to the application of an approximate scaling function the method does not fulfil always the conditions regarding to the compatibility with the partition and the conservation of the piece-wise linearity. VESI meets the requirements 1,4 and 5.

7. CONCLUSIONS

FRI based inference techniques ensure an acceptable output for fuzzy systems even in case of observations when there are no rules whose antecedent part would overlap the observation at least partially.

FSI methods are used for the determination of the shape of the antecedent and consequent sets of the new intermediate rule in the FRI methods that follow the concepts of GM.

This paper surveyed the basic concepts of three FSI methods and evaluated them based on the condition set introduced in section 3.

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