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FUZZY SET APPROXIMATION BY WEIGHTED LEAST SQUARES REGRESSION

[1]Zsolt Csaba JOHANYÁK, [2]Szilveszter KOVÁCS

[1] G.A.M.F. FACULTY, KECSKEMÉT COLLEGE,
DEPARTMENT OF INFORMATION TECHNOLOGY, HUNGARY
[2] UNIVERSITY OF MISKOLC,
DEPARTMENT OF INFORMATION TECHNOLOGY, HUNGARY

ABSTRACT:
Fuzzy rule interpolation based inference techniques are applied in cases when the available rules does not ensure a full coverage for all the possible observations. This paper introduces a new fuzzy set approximation technique (FEAT-WLS) based on three main concepts, namely the virtual shifting of the sets, the calculation of the pieces of the shape by applying a weighted least squares based regression, and obtaining a better set approximation by taking all the sets of the partition into consideration. Its key features are the compatibility with the rule base, the interpolative character, and the conservation of the piece-wise linear shape.

KEYWORDS:
fuzzy rule interpolation, fuzzy set approximation.

1. INTRODUCTION

Interpolative reasoning techniques have been used for several years in order to alleviate the problems characterising the traditional fuzzy reasoning methods (e.g. Zadeh's, Mamdani's) in case of sparse rule bases. These techniques can be divided into two groups depending on their basic concept.

The first one is characterised by the feature that the approximated conclusion is determined directly taking into consideration two or more rules whose antecedent sets surround the observation in each input dimension.

The members of the second group determine that rule first, of which antecedent linguistic terms would be situated in the same position as the sets describing the observation in each dimension. Generally the antecedent of the new rule does not overlap the observation perfectly. Thus this concept requires a second inference step, called single rule reasoning technique, for the determination of the conclusion.

Representative members of the first group are among others the KH method [8] proposed by Kóczy and Hirota, the MACI [10] introduced by Tikk and Baranyi, the FIVE [7] developed by Kovács and Kóczy, the IMUL [11] proposed by Wong, Gedeon and Tikk, and the interpolative reasoning based on graduality introduced by Bouchon-Meunier, Marsala and Rifqi [3]. The structure of the methods belonging to the second group can be described best by the generalized methodology (GM) defined by Baranyi et al. in [1]. Typical members of this group are e.g. the technique family proposed by Baranyi et al. in [1] and the IGRV [4] developed by Huang and Shen.

The rest of this paper is organized as follows. In section 2 we recall the basic concepts of the generalized methodology of the fuzzy rule interpolation. In section 3 a new fuzzy set interpolation method is introduced aiming the determination of the antecedent and consequent parts of the new rule in the first step of the GM. Section 4 outlines some important features of the method through some numerical examples.

2. GENERALIZED METHODOLOGY

The generalized methodology of fuzzy rule interpolation (GM) [1] was introduced by Baranyi et al. Its basic idea is that it divides the task of rule interpolation into two steps. First a new rule is interpolated, of which antecedent part contains linguistic terms, which overlap the fuzzy sets of the observation at least partially and the reference point of each antecedent linguistic term - considering a multidimensional antecedent universe of discourse - has the same abscissa as the respective set of the observation.

The reference point (*RP*) is a point belonging to the shape of a fuzzy set that characterises the position of the set. For example the centre of the core (RP_{cc}), the centre of the support (RP_{sc}) or the centre of gravity (RP_{gc}) can play this role (Fig. 1).

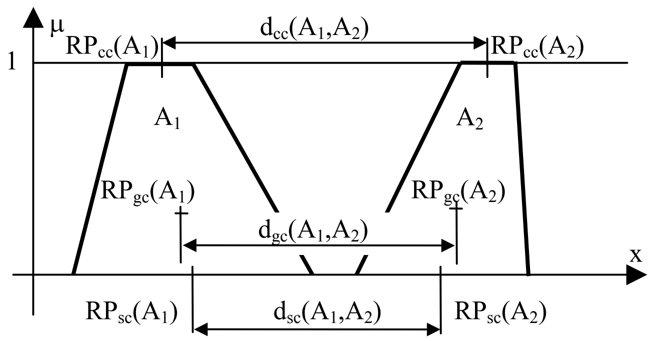

Figure 1. Options for the reference point and the related set distances

The distance of the fuzzy sets ($d(A_1,A_2)$) is measured as a Euclidean distance between their reference points in horizontal direction (Fig. 1). The new rule is determined in three stages. First the antecedent part is calculated using a set approximation or interpolation method. For example the technique SCM [2] or the polar cut based FEAT-p [6] can be used for this task. Secondly the position of the consequent sets is determined e.g. based on the fundamental equation of the rule interpolation (FERI) [8]. Thirdly the shape of the consequent linguistic terms is calculated using the same technique as in the first stage.

In the second step of the GM the conclusion of the inference process is determined by firing the new rule. Usually the antecedent part does not fit perfectly the observation in each dimension and therefore some kinds of special single rule reasoning techniques are needed in order to obtain the desired result. For example the revision principle based FPL, FVL and SRM techniques [9] can be applied in this step.

3. FUZZY SET APPROXIMATION BASED ON LINGUISTIC TERM SHIFTING AND WEIGHTED LEAST SQUARES REGRESSION

The Fuzzy sEt Approximation Technique (FEAT-WLS) being introduced aims the determination of the shape of a new linguistic term at a given position of a fuzzy partition. This position is defined by the abscissa of the reference point of the observation in the j^{th} input dimension ($RP_x(A_j^*)$) in the case of an antecedent partition or by the abscissa of the reference point of the consequence in the k^{th} output dimension ($RP_x(B_k^*)$). This task can be fulfilled by only supposing that there is regularity between the elements of the partition. For the solution of the problem minimum two fuzzy sets are required.

Generally the set approximation techniques seek for the two neighbouring sets that surround the position of the observation (conclusion) and then they interpolate the new set in the given position. We are going out from the assumption that a better set approximation can be attained taking into consideration not only the two flanking sets but all the linguistic terms of a the partition. Moreover in most of the cases due to the small number of the fuzzy sets belonging to the partition it requires less computational complexity to work with all sets.

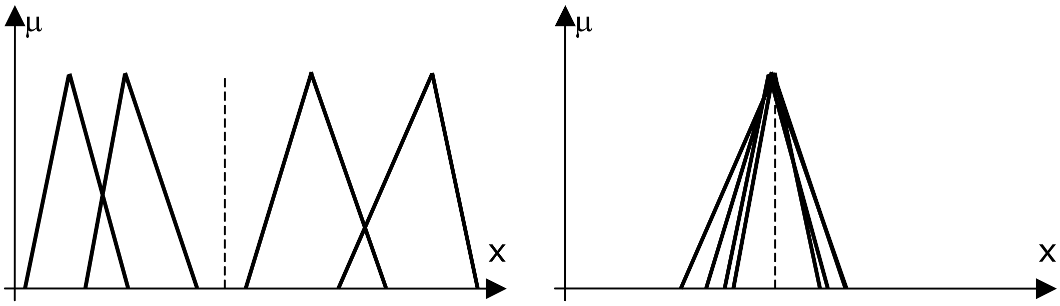

Figure 2. The original partition and the virtually shifted sets

Our technique starts with the determination of the reference point of the linguistic terms of the partition. Next all sets are shifted virtually horizontally into the position of the approximation, i.e. the abscissa of their reference point should coincide with this position. This idea is similar to the concept in [3], but that method uses and translates only the two flanking sets into the location of the observation (conclusion). The left part of fig. 2 presents the original partition, which is sparse in this case for better lucidity. The position of the approximation is indicated by a vertical dashed line. The right part of the figure presents the virtually shifted linguistic terms. The centre of the core was chosen as reference point.

The shape of the new set is determined from the overlapped linguistic terms. There are some possibilities for its calculation. For example the technique FEAT-p [6] introduces the concept of the polar cut by placing a polar co-ordinate system into the projection of the reference point to the abscissa and calculates each point of the shape of the interpolated set as a weighted average of the points situated at the same polar level in the sets taken into consideration.

The advantage of the method is its capability to work with subnormal sets. However, its drawback is that it does not conserve the piece-wise linearity of the fuzzy sets of the partition, which is one of the general conditions introduced in [5] for the evaluation and comparison of the different fuzzy rule interpolation techniques based on the same fundamentals.

Going out from this condition the technique FEAT-WLS (WLS stands for weighted least squares) does not use a point-wise determination of the shape, but assumes from the beginning a polygonal shape of the interpolated set and intends to determine its parameters using a weighted least squares regression based approach. Further on the centre of the core is used as reference point, because it prevents the occurrence of such abnormal interpolated sets whose flanks intersect each other.

The conservation of the piece-wise linearity is only possible when all the sets of the partition possess the same polygonal shape with the break points situated at the same α-levels. Usually the weighted least squares (WLS) are applied in cases when the information provided by the individual data points is not equally precise. In our case the weighting expresses the assumption that the linguistic terms of the partition situated in closer neighbourhood of the observation (conclusion) should exercise a higher influence on the result than the fuzzy sets positioned in farther regions of the partition. The weighting is done by giving each data point its proper amount of influence on the estimated parameters.

Using a WLS based technique for parameter estimation the unknown parameters of the curve describing a part of or the whole shape are estimated by finding the values that minimize the sum of the squared deviations between the theoretical function and the points given by the fuzzy sets taken into consideration at each α-level. It can be described by a WLS criterion function.

The WLS regression can be used with functions, which are linear or non-linear in the parameters. Further on that particular case is examined when the membership functions are piece-wise linear and the breakpoints are situated at the same level in each fuzzy set. For each linear piece – excluding the horizontal parts – of the shape we calculate the criterion function using two α-levels corresponding to the endpoints of the stage.

For example we consider the polygonal shaped linguistic term presented in figure 3. The left part of the figure contains the original partition containing the sets defined by (8), (9), (10) and (11). The position of the approximation is indicated by a vertical dashed line. The right part presents the interpolated set and the characteristic points indicating the shape of the virtually shifted sets. Only the non-horizontal lines are drawn, for which the parameters are going to be estimated.

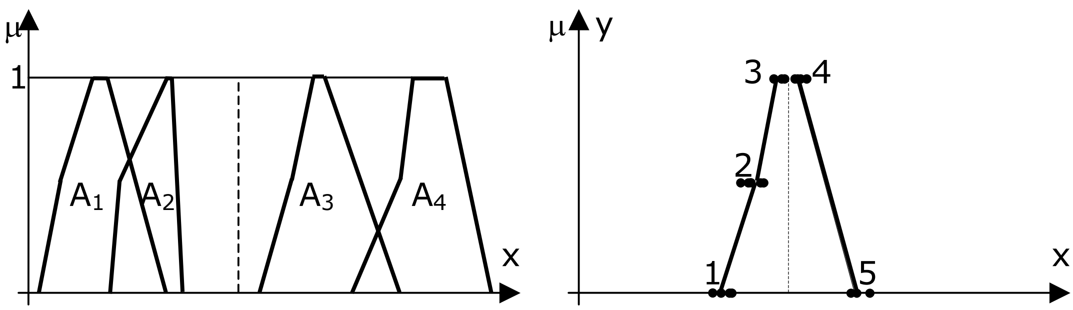

Figure 3. The polygonal shaped linguistic term

The horizontal part (line *34*) will be determined by the endpoints of the other stages. All the sets are convex and normal, and therefore we can assume that $y_1=y_5=0$ and $y_3=y_4=1$. Beside this we define the condition that point 2 should be a real point of the shape, i.e. $y_2\neq 0$ and $y_2\neq 1$. The ordinates of the characteristic points of the interpolated set are pre-determined, therefore the interpolation aims the calculation of the abscissas through the estimation of the parameters of the equations (1), where x_i and y_i are the abscissas and the ordinates of the characteristic points of the approximated set, a_{kl} and b_{kl} are the parameters of the equation that defines the stage with the endpoints *k* and *l*. Using the centre of the core as reference point the left and right flanks can be treated separately. The criterion function for the left flank is given in (2).

$$\begin{aligned} x_1 &= b_{12} \\ x_2 &= a_{12} \cdot y_2 + b_{12} \\ x_3 &= a_{12} \cdot y_2 + b_{12} + (1 - y_2) \cdot a_{23} \\ x_4 &= a_{45} + b_{45} \\ x_5 &= b_{45} \\ b_{23} &= y_2 \cdot (a_{12} - a_{23}) + b_{12} \end{aligned} \tag{1}$$

$$Q_L = \sum_{i=1}^{n} w_i \cdot \left[(b_{12} - x_{1i})^2 + (b_{12} + a_{12} \cdot y_2 - x_{2i})^2 + (b_{12} + a_{12} \cdot y_2 + a_{23} \cdot (1 - y_2) - x_{3i})^2 \right] \tag{2}$$

where n is the number of the sets in the partition and w_i is the weighting factor for the i^{th} set. The parameters a_{kl} and b_{kl} are determined by differentiating Q with respect to each parameter, and setting the result equal to zero. Thus the parameters can be determined by solving the equation system (3).

$$\sum_{i=1}^{n} w_i \cdot \begin{bmatrix} 3 & 2 \cdot y_2 & 1 - y_2 \\ 2 \cdot y_2 & 2 \cdot y_2^2 & y_2 - y_2^2 \\ 1 - y_2 & y_2 - y_2^2 & (1 - y_2)^2 \end{bmatrix} \cdot \begin{bmatrix} b_{12} \\ a_{12} \\ a_{23} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} w_i \cdot (x_{1i} + x_{2i} + x_{3i}) \\ y_2 \cdot \sum_{i=1}^{n} w_i \cdot (x_{2i} + x_{3i}) \\ (1 - y_2) \cdot \sum_{i=1}^{n} w_i \cdot x_{3i} \end{bmatrix} \tag{3}$$

The criterion function (5) and the resulting equation system (4) for the right flank are calculated in a similar way.

$$Q_R = \sum_{i=1}^{n} w_i \cdot \left[(a_{45} + b_{45} - x_{4i})^2 + (b_{45} - x_{5i})^2 \right] \tag{4}$$

$$\sum_{i=1}^{n} w_i \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_{45} \\ b_{45} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} w_i \cdot x_{4i} \\ \sum_{i=1}^{n} w_i \cdot x_{5i} \end{bmatrix} \tag{5}$$

The sets whose original positions were in the neighbourhood of the reference point of the observation (conclusion) should exercise higher influence than those ones situated in farther regions. Therefore the weighting factor is dependent on distance. Its simplest version is the reciprocal value of the distance, which can be expressed by the formula (6) with $p=1$ and $\lambda=1$, but there are several recommendations in the literature for more or less analogue cases. For example in [8] the square of the reciprocal value of the distance is suggested ($p=2$ and $\lambda=1$).

$$w_{jk} = \frac{1}{\lambda \cdot d(A_j^*, A_{jk})^p} \tag{6}$$

$$w_{jk} = e^{-\lambda \cdot d(A_j^*, A_{jk})} \tag{7}$$

where A^*_j is the fuzzy set corresponding to the j^{th} dimension of the observation (conclusion), □ is a positive constant, which in the case of FEAT-WLS can be set to the value *1* because it will fall out during the calculations. The choice of the weighting factor can add a free parameter to the method to adjust the sensitivity. In [12] three variants of the weighting factor are introduced. These can be described by (6) with $p=1$ respective $p=2$ and (7).

In most of the cases interpolation is the desired approximation technique. The term set interpolation (extrapolation) means that in the particular case when the given position is identical with the abscissa of the reference point of an existent set *A* the calculated set also should be identical with *A*.

This task can be fulfilled by setting a condition at the beginning of the algorithm FEAT-WLS, which tests the coincidence. The interpolative approach is also reflected by the weighting factor (6) that assigns an infinite weight to the respective set. Applying interpolation also ensures the condition 4 from [5], namely the compatibility with the rule base, for the rule interpolation method based on FEAT-WLS.

4. NUMERICAL EXAMPLES

In this section we intend to present the effect of the selection of the type and parameters of the weighting factors. We use the partition given in the left part of the figure 3. All linguistic terms are polygonal shaped CNF sets. The breakpoints are situated at the same level.

The characteristic points are given by (8), (9), (10), and (11). In each matrix the first column indicates the abscissas and the second column indicates the ordinates of the characteristic points. The position of the interpolation is $x=0.95$.

Figure 4.a presents the interpolated sets calculated using the weighting factor (6) with $\lambda=1$. Three values were tried for the second parameter. The results are plotted using different line types. Dotted lines indicate $p=1$, dashed lines stand for $p=2$, and continuous lines are used for the case of $p=3$. It can be observed that the increase of the parameter p also entails the increase of the influence of the linguistic term A_2 that is the closest one to the position of the interpolation.

$$A_1 = \begin{bmatrix} 0.05 & 0.00 \\ 0.13 & 0.53 \\ 0.30 & 1.00 \\ 0.37 & 1.00 \\ 0.63 & 0.00 \end{bmatrix} \quad (8) \qquad A_2 = \begin{bmatrix} 0.42 & 0.00 \\ 0.43 & 0.53 \\ 0.64 & 1.00 \\ 0.65 & 1.00 \\ 0.67 & 0.00 \end{bmatrix} \quad (9)$$

$$A_3 = \begin{bmatrix} 1.07 & 0.00 \\ 1.20 & 0.53 \\ 1.27 & 1.00 \\ 1.37 & 1.00 \\ 1.68 & 0.00 \end{bmatrix} \quad (10) \qquad A_4 = \begin{bmatrix} 1.47 & 0.00 \\ 1.68 & 0.53 \\ 1.73 & 1.00 \\ 1.90 & 1.00 \\ 2.10 & 0.00 \end{bmatrix} \quad (11)$$

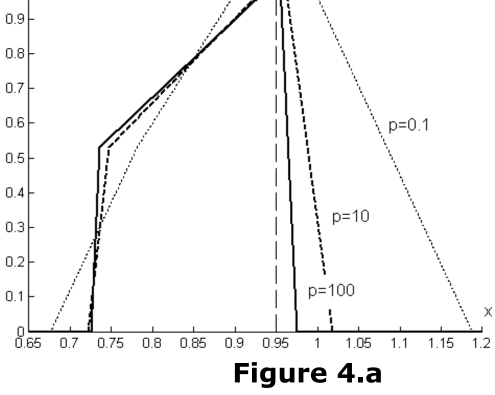

Figure 4.a

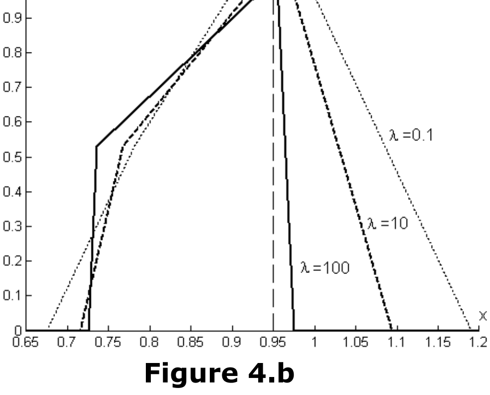

Figure 4.b

Figure 4.b contains the interpolated sets using weighting factor (7). For the parameter λ also three values were tried. The results are indicated by different line types. Similarly to the previous weight type by increasing the value of the parameter the closest set (A_2) becomes more and more dominant.

6. CONCLUSIONS

Fuzzy rule interpolation based techniques have been proved to be a prosperous solution in case of systems applying sparse rule bases. This paper introduced a new set approximation technique called FEAT-WLS, which is applicable in the first step of the generalized methodology of fuzzy rule interpolation [1]. The basic concepts of the method are the virtual shifting of the linguistic terms in order to reach the coincidence between their reference points and the position of the approximation, and the determination of the elements of the shape applying a weighted least squares based regression.

The most important advantages of the FEAT-WLS are that it conserves the piece-wise linear character of the shape of the linguistic terms of the partition and secondly, in case of a coincidence between the position of the approximation and the reference point of the set, it generates the approximated set to be identical with the corresponding original one.

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