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Interpolation-based Fuzzy Reasoning - a comparison

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This paper reviews some important points of sparse rule-bases, the reason of their generation and after that three methods are presented, which allow the approximation of the missing rules with reasonable demand on computing. The delimitations and advantages of these methods are presented, too.

SOME IMPORTANT QUESTIONS OF SPARSE RULE-BASES AND THE REASONS OF THEIR GENERATION

Fuzzy systems based on a sparse rule-base do not have rules for all the possible combinations of observations. Thus a system working with classical fuzzy reasoning e.g. based on Compositional Rule of Inference can fire none of the rules by some observed values and will have no output.

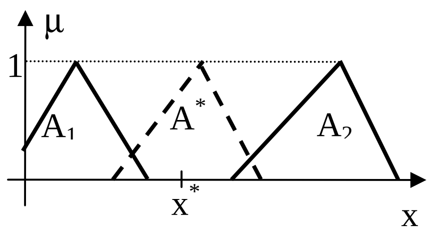

Fig. 1.

As an example let us see a system having an input linguistic variable with a partition as it can be seen in Fig. 1. There are rules for the linguistic terms A_1 and A_2, but there is no rule for the fuzzy set A* marked by dashed lines. In the case of an observation of x=x* and the lack of a rule with matching antecedent part the system can not produce an output.

How can a sparse rule-base emerge?

Essentially a sparse rule-base takes its origin from one of the three reasons specified below:

1. The rules generated from information obtained from experts or from other sources (e. g. neural network-based learning techniques) do not cover all the possible observation values. For instance assuming the partition in Fig. 1. on a

one dimensional universe of discourse the rule-base only contains elements that have only the fuzzy sets A_1 or A_2 as antecedents.

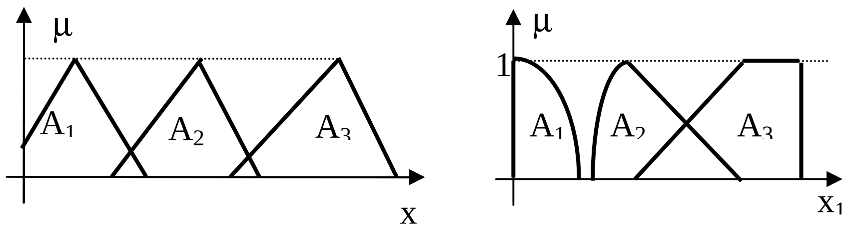

Fig. 2.

2. Gaps between the fuzzy sets can be arisen during the fine-tuning of the system due to the modification of the shape of membership functions (Fig. 2.).
3. The number of the linguistic variables is so high that even if all the possible rules can be found out they could not be stored under the given hardware conditions. For instance assuming an observation with n=3 dimensions and k=5 linguistic terms for each variable and supposing that in the antecedent parts of the rules only the AND relations are allowed and all antecedents contain each dimension, k^n=125 rules would be needed. Taking no notice of the conditions mentioned above the number of the rules grows on. The great number of the rules increases the duration of the inference, too. Thus the performance of the system is decreasing. Making a rule-base sparse artificially could be a possible solution for such cases.

METHODS APPLICABLE IN CASE OF SPARSE RULE-BASES

In the lack of coverage by some observations methods based on rule approximation for turning out the consequence set should be applied. As a first step should be assured that each input linguistic variable has a ε-covering partition, where ε should be greater than zero. It can be fulfilled by introducing new linguistic terms. Observing a value without any rule in the course of system operation produces a new rule considering the existing rules in the neighbourhood of the observation. The principle is the more similar an observation to an antecedent part of a rule is the better it should resemble the estimated result of the consequent part of that rule [1].
The condition of the approximation is that there should exist at least a partial ordering relation over the fuzzy sets occurring in the antecedent and consequent parts of the rules [2]. First the similarity of fuzzy sets should be defined to solve the problem. This can be determined by evaluating the distance of the sets.

THE DISTANCE OF FUZZY SETS

The precedence relation is defined by fuzzy sets with the help of α-cuts. Let the A and B fuzzy sets be normal and convex and let inf and sup be the infinum respective the supremum of an α-cut. If for $\forall\ \alpha \in (0,1]$ the conditions $\inf\{A_\alpha\} < \inf\{B_\alpha\}$ and $\sup\{A_\alpha\} < \sup\{B_\alpha\}$ are hold, A<B is fulfilled. The distance of two fuzzy sets is expressed by means of a fuzzy set which is defined over the interval [0,1]. In the

course of calculations the Euclidean distances between the end points of the α-cuts are considered. These are called lower (d_L^α) and upper (d_U^α) distances and are calculated by formulas (1) and (2).

$$d_L^\alpha(A_1, A_2) = \inf\{A_2^\alpha\} - \inf\{A_1^\alpha\} \quad (1)$$
$$d_U^\alpha(A_1, A_2) = \sup\{A_2^\alpha\} - \sup\{A_1^\alpha\} \quad (2)$$

In the following sections three important interpolation-based methods are presented which can be used during the rule approximation.

KÓCZY AND HIROTA'S METHOD

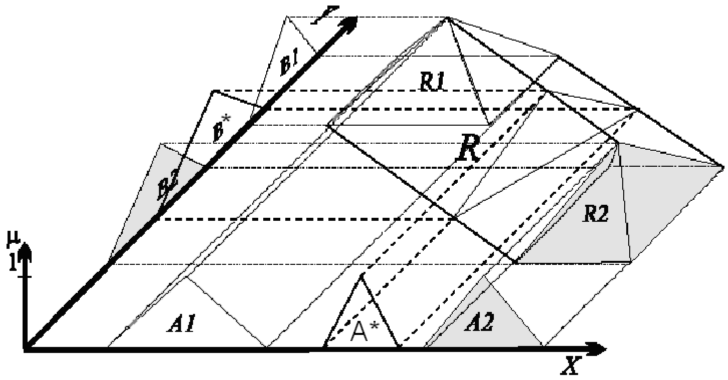

Fig. 3.

An important precondition of applying the K&H method is that we should find at least two rules surrounding the observation. Let us denote with B_i (B_1, B_2, etc.) the linguistic terms of the output linguistic variable. If one is using distance-based similarity measure, the basic idea of the rule approximation is that the closer the x observation belonging to the A* fuzzy set to the antecedent A_i (A_1, A_2, etc.) is, the closer the consequent set of the estimated rule should be to the B_i consequent. This expectation can be fulfilled proportioning the distances [2].
The rule-system shown in Fig. 1. contains two rules. Each of them contains only one linguistic term in its antecedent and consequent part. The rule-base is sparse, therefore in case of an input value between A_1 and A_2 a rule approximation is needed for the inference. Building the distance proportions (3) and using the (1) and (2) formulas in the case of each α-cut the lower (4) and upper (5) points of the consequence fuzzy set can be determined.

$$\frac{d_i^\alpha(A_1, A^*)}{d_i^\alpha(A^*, A_2)} = \frac{d_i^\alpha(B_1, B^*)}{d_i^\alpha(B^*, B_2)} \quad (3)$$

where i can be L or U depending on whether lower or upper point was calculated.

$$\inf\{B^{*\alpha}\} = \frac{d_L^\alpha(A_1, A^*) \cdot \inf\{B_2^\alpha\} + d_L(A^*, A_2) \cdot \inf\{B_1^\alpha\}}{d_L^\alpha(A_1, A^*) + d_L^\alpha(A^*, A_2)} \quad (4)$$

$$\sup\{B^{*\alpha}\} = \frac{d_U^\alpha(A_1, A^*) \cdot \sup\{B_2^\alpha\} + d_U^\alpha(A^*, A_2) \cdot \sup\{B_1^\alpha\}}{d_U^\alpha(A_1, A^*) + d_U^\alpha(A^*, A_2)} \tag{5}$$

The needed consequence fuzzy set is determined in resolution form as a union of α-cuts. The K&H linear interpolation only works efficient if the shape of the antecedent fuzzy sets are simple, possibly piecewise linear (e.g. triangle). Fulfilling this condition makes possible the description of the sets with only a few characteristic points. Thus it can be achieved that the calculations have to be made only for the significant α-cuts.

The benefits of the above presented interpolative method are the easy interpretability and feasibility as well as the low computational complexity. Its drawback is that it can be applied only in case of accomplishment of the conditions given in (6) and (7) [3]. In other cases it can give birth of abnormal fuzzy sets.

$$\frac{d_L^\alpha(A_1, A^*)}{d_L^\alpha(A_1, A^*) + d_L^\alpha(A^*, A_2)} = \beta \in [0,1] \tag{6}$$

$$\frac{d_L^\alpha(B_2^\alpha, B_1^\alpha)}{d_L^\alpha(A_2^\alpha, A_1^\alpha)} = \gamma > 0 \tag{7}$$

THE LINEAR INTERPOLATION-BASED METHOD PROPOSED BY HSIAO, CHEN AND LEE [3]

The author's aim was the development of an approximation method which guarantees the triangular form of the consequence set in case of antecedents having triangular shapes. The method is based on K&H interpolation. As a first step the base points of the consequence are determined at a given α level (generally α=0) with the K&H method and after that the highest point is calculated.

Further on only the solution of this latter task is presented. Let us notate with k_i and t_i (i=1,*,2) the left and right slopes of the three linguistic terms in Fig. 1. Further on let us notate with h_i and m_i (i=1,*,2) the left and right slopes of the three linguistic terms of the output linguistic variable and let

$$k_* = k_1 x + k_2 y \tag{8}$$

$$t_* = t_1 x + t_2 y \tag{9}$$

where x and y are real numbers. x and y can be determined from the above two equations supposing the proportion of the slopes for each set is different. Let us notate with $hst\{B_\alpha^*\}$ the value of the universe of discourse corresponding to the highest point of the estimated set. In this case the slopes of the consequence can be expressed by the formulas given in (10) and (11). From these the value of $hst\{B_\alpha^*\}$ (12) can be determined.

$$h_* = |h_1 x + h_2 y| c = \frac{1 - \alpha}{hst\{B_\alpha^*\} - \inf\{B_\alpha^*\}} \tag{10}$$

$$m_* = -\left|m_1 x + m_2 y\right| c = \frac{\alpha - 1}{\sup\left\{B_\alpha^*\right\} - hst\left\{B_\alpha^*\right\}} \quad (11)$$

$$hst\left\{B_\alpha^*\right\} = \frac{m\left(\sup\left\{B_\alpha^*\right\}\right) - h\left(\inf\left\{B_\alpha^*\right\}\right)}{m - h} \quad (12)$$

One of the advantages of this method is that it generates normal fuzzy sets on such occasions when the conditions of the K&H interpolation are not met. It has low computational pretensions and is well applicable when the known membership functions have triangular forms. As disadvantage can be mentioned that this method is not generally usable for all the membership function types.

THE MODIFIED α-CUT BASED INTERPOLATION

The method MACI developed by Tikk and Baranyi [5] uses the vector representation of fuzzy sets suggested by Yam and Kóczy in [4]. For instance the fuzzy set in Fig. 4. is an isosceles triangle, which can be given through three points ($[a_{-1}, a_0, a_1]^{-1}$), which are called characteristic points.

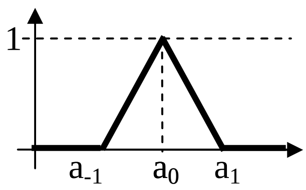

Fig. 4.

During the short presentation of this method we are only dealing with the upper edge ($[a_0, a_1]^{-1}$), the lower edge can be handled similarly. The points are referred to by two indexes with the notation a_{ij}. The first one (i) is the number of the rule. In our example this can be 1 or 2. The second one is the serial number of a characteristic point of the fuzzy set determined by the first index. In our case it could be 0 or 1. Thus $\underline{a}_1=[a_{10}, a_{11}]^{-1}$ is the vector describing the antecedent part of that rule which is the left side one from the nearest two rules wich surround the observation. The consequent fuzzy sets are described in a similar way, e.g. in the case of the first rule $\underline{b}_1=[b_{10}, b_{11}]^{-1}$.

During the interpolation of the consequence belonging to the observation x a coordinate transformation is made. It happens on the purpose to avoid the possibility of abnormal reasoning. The non-negative result and the monotonous increasing coordinate values of the consequence are assured, too. The vector describing the upper edge is given by the formulas shown below in case of the approximation of the consequent fuzzy set by a triangular shape.

$$\underline{y} = \underline{y}' \cdot \underline{\underline{T}}^{-1} \quad (8)$$

$$\underline{\underline{T}}^{-1}=\begin{bmatrix}\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & 0\end{bmatrix} \tag{9}$$

$$\underline{y}'=\left(\underline{\underline{I}}-\underline{\underline{I}}\cdot\underline{\Lambda}\right)\cdot\underline{b}_1'+\underline{\underline{I}}\cdot\underline{\Lambda}\cdot\underline{b}_2' \tag{10}$$

$$\underline{\underline{I}}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{11}$$

$$\underline{b}_1'=\left[b_{10}\cdot\sqrt{2}\quad b_{11}-b_{10}\right]^{-1} \tag{12}$$

$$\underline{b}_2'=\left[b_{20}\cdot\sqrt{2}\quad b_{21}-b_{20}\right]^{-1} \tag{13}$$

$$\underline{\Lambda}=[\lambda_0\quad\lambda_1] \tag{14}$$

$$\lambda_0=\frac{x_0-a_{10}}{a_{20}-a_{10}} \tag{15}$$

$$\lambda_1=\frac{x_1-a_{11}}{a_{21}-a_{11}} \tag{16}$$

The method is well suitable for the case of complex shaped membership functions, too. Its advantages can be summarized in the following points.

- The computational time is not increased in comparison with the basic interpolation type.
- The conclusion preserves the piecewise linearity for the intervals between the characteristic points [2] with a good approximation.
- The abnormal results are avoided.

CONCLUSION

Fuzzy systems using classical reasoning methods can not produce an output for each possible input value when their rule-base is sparse. A sparse rule-base takes its origin from insufficient information or from certain steps of system development. All the three presented linear interpolation-based rule approximation methods are easy to put into practice and have a low need on computational time. The most advantageous among them is the modified α-cut based interpolation.

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