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Polar-cut based fuzzy model for petrophysical properties prediction

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Abstract ***– The application of fuzzy rule interpolation (FRI) methods in fuzzy models can reduce the complexity of the fuzzy model significantly. In case of automatic model generation this reduced complexity also leads to quicker convergence of the fuzzy model.***

The goal of this paper is the detailed investigation of a fuzzy model construction in a real world problem, i.e. the prediction of petrophysical properties, which is an important supporting tool in taking decisions on rentability of the exploration of a specific region.

Keywords: ***fuzzy rule interpolation, automatic rule base generation, FRIPOC, RBE-DSS.***

I. INTRODUCTION

The popularity of Fuzzy Rule Interpolation (FRI) based fuzzy models is emerging nowadays. The reason is very simple. The common "fuzzy dot" (or fuzzy relation) representation of fuzzy rules in fuzzy models, in case of classical fuzzy reasoning methods (e.g. the Zadeh-Mamdani-Larsen Compositional Rule of Inference (CRI) Zadeh [37], Mamdani [25], Larsen [24], or the Takagi - Sugeno fuzzy inference [29]) are assuming the completeness of the fuzzy rule base and hence, the exponential complexity of the fuzzy model with respect to its input space dimensions.

The appearance of FRI methods in fuzzy models where the derivable rules are deliberately missing from the fuzzy model (i.e. the model rule base is "sparse") can reduce the complexity of the fuzzy model significantly. The reduced complexity of the fuzzy model also leads to quicker convergence in case of automatic fuzzy model generation based on known input-output pairs. On the other hand the FRI methods and the FRI based fuzzy models are considered to be novelties and they are still under investigation.

As a validation of the practical applicability of the fuzzy model identification method RBE-DSS [15] with the fuzzy inference method FRIPOC [10] we chose a real world problem inherited from the field of well log analysis.

The rest of this paper is organized as follows. Section II recalls the concepts of dense and sparse rule bases. Section III presents the main ideas of the polar cut based FRIPOC inference method. The basic concepts of the applied fuzzy model identification method are presented in section IV followed by the short description of the modeled problem in section V. Section VI deals with the results of the modeling comparing the obtained system structure and performance with the previously published ones.

II. FUZZY SYSTEMS BASED ON SPARSE RULE BASES

A. Sparse vs. dense rule base

The first developed fuzzy reasoning methods (classical methods) require a full coverage ($\varepsilon > 0$ in (1)) of the input space by the antecedent sets of the rules

$$\arg\max_{\varepsilon}\left(\min_{i=1}^{N}\left\{\max_{j=1}^{n_i}\left\{t\left(A_{i,j},A_i^*\right)\right\}\right\}\geq\varepsilon,\forall A_i^*\subset X_i\right),\quad \varepsilon\in[0,1], \tag{1}$$

where N is the number of antecedent dimensions, n_i is the number of fuzzy sets from the i^{th} antecedens dimension participating in rule antecedents, A_i^* is the observation set in the i^{th} antecedent dimension and $A_{i,j}$ is the j^{th} set in the i^{th} antecedent dimension.

This feature is called dense character of the rule base. If a rule base is dense it contains for each allowed observation value at least one rule whose antecedent sets overlap or intersect the observation in each input dimension.

Fig. 1. illustrates the antecedent space of a dense rule base. The system consists of three input dimensions. Each rule antecedent is represented by a cube defined by the supports of the sets participating in a rule. For the sake of simplicity

in this case each antecedent partition contains three fuzzy sets.

In case of dense rule bases the number of rules increases exponentially with the number of input dimensions and the number of linguistic terms in the partitions. In order to eliminate the drawbacks of increasing complexity, Kóczy and Hirota [21] suggested first the application of the so called sparse rule bases. When a rule base is sparse it is not ensured the full coverage of the input space, i.e. $\varepsilon=0$ in (1). Fig. 1. illustrates a sparse rule base containing only two rules.

B. Reasoning in sparse rule bases

Due to the lack of coverage in sparse rule bases the classical reasoning methods like Zadeh's [37], Mamdani's [25], Takagi-Sugeno's [29], etc. cannot always afford an acceptable output. Therefore in case of these systems special approximate reasoning methods have to be used for the calculation of the conclusion. They may be used in case of fuzzy control [28] as well.

Usually methods based on fuzzy rule interpolation (FRI) are applied in such cases. Their research and development was started by Kóczy and Hirota who created the first FRI technique called linear rule interpolation or KH method [21]. Since then quite a significant amount of literature has been published on this field. Overviews and evaluations of the available methods can be found in [7], [23], [26], and [27].

III. FUZZY REASONING BY THE FRIPOC METHOD

We used the FRIPOC [10] fuzzy rule interpolation method in course of the fuzzy model identification. Its main idea is that it determines the shape of the conclusion by its polar cuts. The method follows the concepts of the generalized methodology of fuzzy rule interpolation (GM) developed by Baranyi, Kóczy and Gedeon [1]. It identifies the position of the fuzzy sets by their reference points using the midpoint of the core as reference point (Fig. 2). The conclusion is determined in two steps.

First an auxiliary rule is interpolated whose antecedent sets are in the same position as the current input sets (observation) in each antecedent dimension, i.e. in each

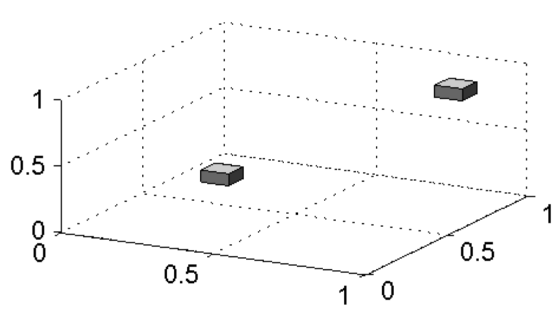

Fig. 1. Antecedent space of the raw fuzzy system

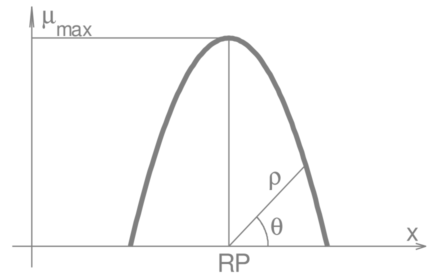

Fig. 2. Reference point and polar cut

input dimension the reference point of the observation will be identical with the reference point of the antecedent set of the new rule.

The antecedent and consequent sets of the new rule are determined by set interpolation using the FEAT-p [14] technique. The position of the consequent sets is calculated by an adapted version of the Shepard interpolation [30].

One calculates the shape of the conclusion in the second step by the SURE-p [10] method using the observation and the interpolated rule.

A. Polar cut

The FEAT-p and SURE-p methods use polar cuts in course of the fuzzy sets' shape calculation. The concept of polar cuts is based on the application of a polar coordinate system that is placed at the reference point of the set (Fig. 2).

A polar cut describes one point on the shape of a fuzzy set. It consist of a value pair $\{\rho,\theta\}$, where ρ is the polar distance of the point and θ is the corresponding polar angle (Fig. 2).

Similar to the case of α-cuts an extension and resolution principle can be formulated for polar cuts as well stating that each convex fuzzy set can be decomposed into polar cuts and can be composed from polar cuts.

B. The FEAT-p method

The Fuzzy sEt interpolATion based on polar cuts [14] aims the creation of a new fuzzy set in a given point of a universe of discourse called interpolation point. The calculations are based on a supposed regularity between the known sets of the partition. It also uses the concept of Linguistic Term Shifting (LTS) [13] and polar cuts.

Conform to LTS, first all known sets of the partition are shifted horizontally in order to reach the coincidence between their reference points and the interpolation point. Next the shape of the new set is calculated by its polar cuts. Each polar length is determined as a weighted average of the corresponding polar lengths of the overlapping known sets.

The weighting is based on the original distance of the sets from the interpolation point. Some applicable weight functions are presented in [14].

C. The SURE-p method

The antecedent sets of the interpolated rule generally are not identical with the observation sets in all input dimensions. Therefore the conclusion is determined by a special single rule reasoning method that modifies the consequent sets of the new rule taking into consideration the similarity between the observation and the rule antecedent sets.

The Single rUle REasoning based on polar cuts [10] first calculates for each polar cut and for each input dimension the difference between the polar length of the observation and the polar length of the rule antecedent set. Next an average difference is determined for each polar level and the consequent polar lengths are modified by this resulting difference.

The final shape of the conclusion fuzzy set is determined by a control and correction algorithm that ensures the avoidance of the abnormal set shapes.

IV. MODEL IDENTIFICATION

Fuzzy model identification aims the determination of the antecedent and consequent fuzzy sets and the rule base that describes the relation between the input and output of the system.

Due to its advantages regarding the reduced system complexity, the application of methods that result in sparse rule bases seemed to present the best solution. Generally there are two main ways to produce a rule base with reduced number of rules. The first one starts with a dense rule base and reduces the number of rules by either eliminating the rules considered as non-relevant or merging the nearest (similar) rules (e.g. solutions based on evolutionary algorithms were proposed by Botzheim, Cabrita, Kóczy and Ruano [2], by Botzheim, Hámori and Kóczy [3], as well as by Kóczy, Botzheim and Gedeon [20]).

The methods following the second way create directly a rule base that not ensures a full coverage of the input space. One can distinguish here three different approaches. The first one endeavors to identify the so called optimal fuzzy rules [17][18]. The second one generates partitions and rules applying fuzzy clustering [4][5][16][31][32][36].

A. The RBE-DSS method

The Rule Base Extension with Default Set Shapes (RBE-DSS) [15] was developed for the identification of MISO and SISO fuzzy models from sample data. However, it also can be applied in case of multiple output phenomenon by generating separate rule bases for each output dimension.

The basic idea of the method is that one creates first a starting rule base containing only two rules, which describe the minimum and the maximum output of the modeled process (see fig. 1). Next an iterative tuning algorithm is started that applies a hill climbing approach by modifying with a predefined step each parameter value one-by-one in both the lower and upper directions and testing the system in case of each new parameter value against the sample data set.

If the amelioration of the system performance slows down or even stops the method generates a new rule in order to improve the system. The new rule is created in that point where the difference between the prescribed output and calculated output of the system is maximal. One calculates the shapes of the new linguistic terms using so called default set shapes, which means that for each partition a prototype fuzzy set is defined at the beginning. Its type and parameters depend on the characteristic shape type and the width of the partition.

The model identification stops when either the performance index becomes better than a prescribed threshold value or the preset iteration limit is reached.

B. Performance index

After each parameter adjustment the resulting parameter set is evaluated by calculating the system output for a collection of predefined input data, for which the expected output values are known. In order to compare the results obtained with different parameter sets a performance index is calculated after each system evaluation.

There are several applicable performance index types [6]. We used the relative value of the root mean square (quadratic mean) of the error (RMSEP) as performance index for the evaluation of the fuzzy system. We chose it owing to its easy interpretability. The value of the root mean square (quadratic mean) of the error [34] is calculated by

$$RMSE = \sqrt{\frac{\sum_{j=1}^{M} \left(y_j - \hat{y}_j\right)^2}{M}}, \qquad (2)$$

where M is the number of training data points, y_j is the output of the j^{th} data point and $\hat{y}_j$ is the output calculated by the system.

The relative value of *RMSE* to the range (*RMSEP*) expressed in percentage is determined by

$$RMSEP = \frac{RMSE}{DR} \cdot 100, \tag{3}$$

where DR is the range of the output dimension.

V. PETROPHYSICAL PROPERTIES

One of the key tasks in course of the analysis of petroleum well log data is the prediction of petrophysical properties corresponding to specific input data, i.e. depth values different from the original ones used by the experiments. Such properties are the porosity, permeability and volume of clay [35]. The expensive and time consuming character of the data collection from boreholes increases the significance of the prediction. The predicted values help taking decisions on rentability of the exploration of a specific region.

Our research task was to create a fuzzy model with low complexity that is applicable for the prediction of porosity (PHI) based on well log data described by three input variables. These are the gamma ray (GR), deep induction resistivity (ILD) and sonic travel time (DT).

VI. EXPERIMENTAL RESULTS

For the sake of easier comparability with the previously published results ([35] and [8]), the same training and testing data sets were used as it was introduced in [35]. The training data set consisted of 71 data points and the testing data set consisted of 51 data points. The data were preprocessed and each variable was normalized to the unit interval.

In course of the model identification and testing we used the Fuzzy Rule Inerpolation (FRI) and RuleMaker Matlab toolboxes. Both of them are available under GNU GPL from [38].

The FRI ToolBox [12] contains the implementation of eleven interpolation based fuzzy inference methods and supports the testing of fuzzy systems as well as the 2D and 3D visualization of the fuzzy sets and rule base.

Our RuleMaker ToolBox supports the automatic fuzzy model identification from input and output sample data. The user can choose fuzzy clustering or RBE based approaches, can apply four parameterization modes and seven performance indices. The performance of the system and the results are visualized using graphical and alphanumerical output.

The applied inference technique was FRIPOC [10] combined with the COA defuzzification and we used RBE-DSS [9] for system generation and tuning. The antecedent and consequent partitions of the final system are presented on figures 5, 5, 7, and 7. The rule base of the tuned system is sparse. Fig. 4. visualizes the antecedent space of the rule base each rule antecedent being represented by a cube defined by the supports of the antecedent sets of the rules.

The fuzzy system was generated using RMSEP as performance index. In order to compare the results with those published in [35] we also evaluated the final system against the training and testing data set by the correlation factor (4), which was used in [35] as a prediction accuracy indicator.

$$R = \frac{\sum_{j=1}^{M} \left(y_j - \bar{y}\right) \cdot \left(\hat{y}_j - \bar{\hat{y}}\right)}{\sum_{j=1}^{M} \left(y_j - \bar{y}\right)^2 \cdot \sum_{j=1}^{M} \left(\hat{y}_j - \bar{\hat{y}}\right)^2} \tag{4}$$

In case of the training data set the new system showed a better performance than the previously published ones. However in case of the testing data the correlation between the prescribed output and the calculated one was slightly worser than in case of the system applying the LESFRI inference method. Table *1*. presents the correlation factor values obtained after the evaluation of our system and those published in [35] and [8].

An advantageous feature of the new system is that the number of linguistic terms and rules is significantly reduced in comparison to [35]. For example while the system presented in [35] was based on 63 rules, our version contains only 37 rules. However, it should be mentioned as a drawback that the cost of the better performance index was an increase in the number of the rules and linguistic terms compared to the system described in [8].

Besides, as a result of the improvement of the tuning algorithm the shapes of the membership functions of the

TABLE 1. Correlation factor values

Applied method	Correlation Factor	
	Training data	Testing data
MACI [35]	0.917	0.865
RBE-DSS + LESFRI [8]	0.934	0.890
RBE-DSS + FRIPOC	0.966	0.880

TABLE 2. Number of fuzzy sets in the input and output dimensions

Applied method	GR	FS	DT	PHI	Σ
[35]	6	4	4	5	19
[8]	8	7	9	7	31
Curr.	13	8	11	7	39

input and output partitions (presented on fig. 5, 5, 7, and 7.) are more nice and interpretable than those in [8]. However, they are not so uniform as the triangle shaped ones introduced in [35].

Another advantage of the current solution as well as of the one presented in [8] is that owing to the application of the fuzzy rule interpolation methods FRIPOC and LESFRI the fuzzy system is able to produce an interpretable output for each possible input value. This property was nnot ensured in case of system [35].

III. CONCLUSIONS

This paper presented a practical application of the method pair RBE-DSS and LESFRI. The automatic fuzzy model identification was successfully applied in case of a real world problem, the prediction of porosity (PHI) from well log data knowing three measured values the gamma ray

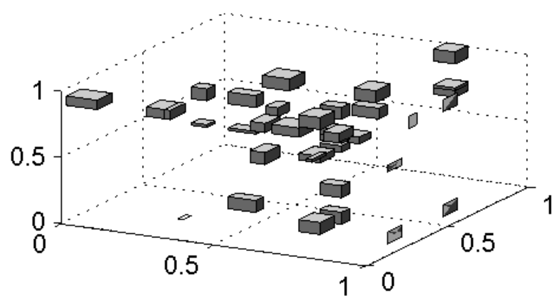

Fig. 5. Antecedent space of the tuned fuzzy system (sparse rule base)

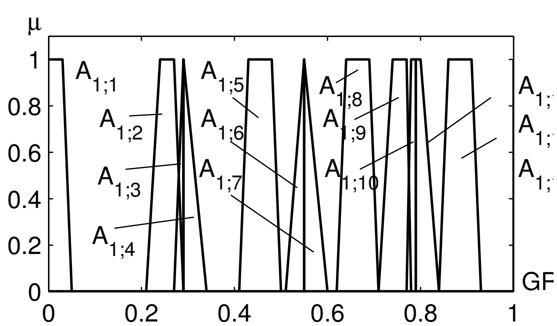

Fig. 5. Antecedent partition of the tuned system for gamma ray (GR)

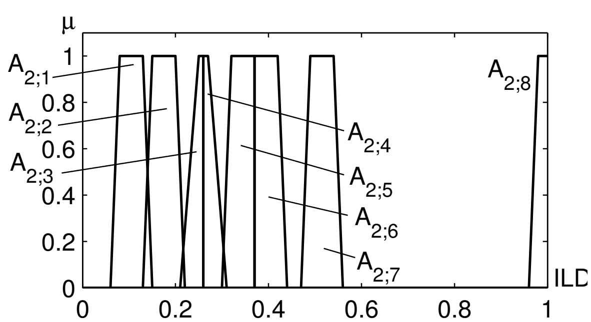

Fig. 5. Antecedent partition of the tuned system for deep induction resistivity (ILD)

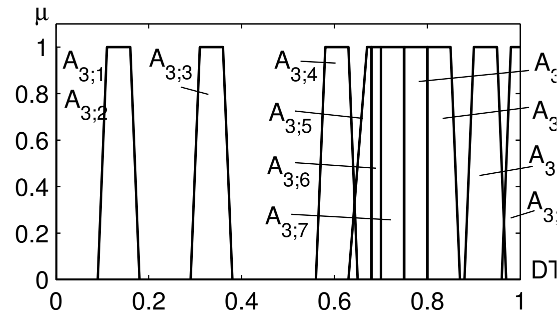

Fig. 7. Antecedent partition of the tuned system for sonic travel time (DT)

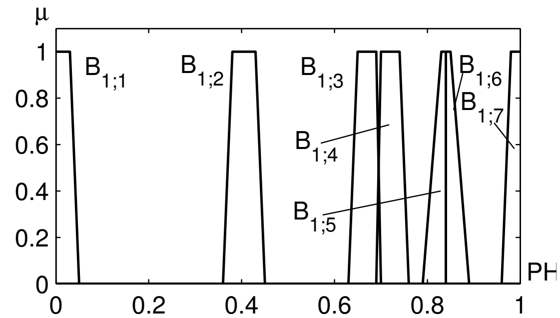

Fig. 7. Consequent partition of the tuned system (porosity - PHI)

(GR), the deep induction resistivity (ILD) and the sonic travel time (DT).

The resulting system showed a better performance index than the previously published solutions ([35] and [8]). This result was achieved with rule number that was between the rules required by the previous solutions. Future research will be focused on the stability analysis [33] of the generated systems.

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