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SIMILARITY MEASUREMENT IN INTERPOLATIVE FUZZY REASONING

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Abstract: Systems based on interpolative fuzzy reasoning work with sparse rule bases. In case of some input values the system should approximate the output value. Carrying out this task depends on the right selection of the suitable fuzzy similarity measure. The goal of this paper is presenting two of such measures, which are also applied in some interpolation based fuzzy reasoning methods.

Key words: similarity measure, distance of fuzzy sets, vague environment

1 Introduction

The functioning of reasoning systems working with fuzzy logic is based on rules. If a system in case of an input value (observation) does not dispose of at least one rule whose antecedent part contains a linguistic term whose support contains the observed value, the system should approximate the output value taking into consideration the existent rules. The adoption of interpolative techniques is the best solution for such cases. The similarity measurement of non overlapping or non intersecting fuzzy sets and the similarity measurement of a fuzzy set and a singleton are one of the fundamental questions of the fuzzy interpolation. The most obvious way of calculating this similarity is based on distance measurement.

There are a lot of distance measures in the literature, which are more or less suitable for different tasks in fuzzy logic. In terms of the interpolation a method can be considered as advantageous if it has low computational complexity and the information describing the shape of the membership function is not lost during the calculations.

In other terms a distance measure would be useful, which could give the chance of the reconstruction of a fuzzy set from another set and from their distance, at least in the one dimensional case. In this paper we would like to introduce two of such measures, which are also applied in some interpolation based fuzzy reasoning methods.

2 Fuzzy distance

The fuzzy distance introduced by Kóczy in [1] fulfils the requirements specified in previous section. It is based on the α-cuts of the two fuzzy sets. It is expressed by means of a fuzzy set which is defined over the interval [0,1]. In the case of calculations the Euclidean distances between the end points of the α-cuts are considered. These are called lower (d_L^α) and upper (d_U^α) distances and are calculated by formulas (1) and (2). A fuzzy partition containing two non overlapping triangle shaped fuzzy sets is presented on the left side of Fig. 1. The lower and upper distances are indicated for the level α=0.6. On the right side the lower and upper distances are plotted against α.

$$d_L^\alpha(A,B)=\inf\{B^\alpha\}-\inf\{A^\alpha\} \tag{1}$$

$$d_U^\alpha(A,B)=\sup\{B^\alpha\}-\sup\{A^\alpha\} \tag{2}$$

If the universe of discourse is multi-dimensional, the distances between $\inf\{A_\alpha\}$, $\inf\{B_\alpha\}$ and $\sup\{A_\alpha\}$, $\sup\{B_\alpha\}$ can be defined in the Minkowski sense:

$$d_L^\alpha(A,B)=\left(\sum_{i=1}^{k} \mathrm{d}_\mathrm{L}^\alpha(\mathrm{A_i},\mathrm{B_i})^\mathrm{w}\right)^{1/\mathrm{w}} \tag{3}$$

$$d_U^\alpha(A,B)=\left(\sum_{i=1}^{k} \mathrm{d}_\mathrm{U}^\alpha(\mathrm{A_i},\mathrm{B_i})^\mathrm{w}\right)^{1/\mathrm{w}} \tag{4}$$

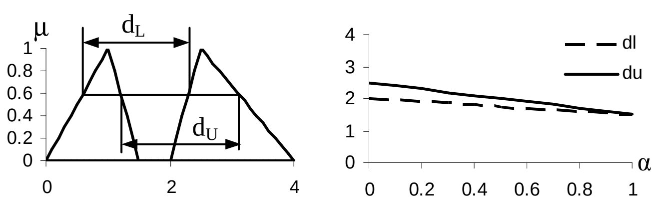

Fig. 1. Non overlapping fuzzy sets and their Kóczy distance

An important requirement for the existence of the fuzzy distance is that all the comparable fuzzy sets should be convex and normal, otherwise some α-cuts are not connected or do not exists at all, which makes the distance corresponding to these α-cuts meaningless. The only disadvantage of using the fuzzy distance for interpolative fuzzy reasoning is that it is little bit difficult to handle. The interpolation method proposed by Kóczy and Hirota in [5] is based on the fuzzy distance and it is a fuzzy extension of the classical linear interpolation. It is called the fundamental equation of fuzzy rule interpolation [4]. The consequence fuzzy set is determined in resolution form as a union of α-cuts, which are calculated by means of the Shepard interpolation. The weak point of the α -cut based interpolation is that we should find the two nearest rules which surround the observation and in the case of arbitrary shaped convex normal fuzzy sets theoretically an infinite number of α-levels should be taken into account for a proper conclusion [4], which could be a time consuming task.

3 Distance measurement in vague environment

Klawonn introduced a new approach to handle imprecise numbers in [2]. He used the concept of vague environment, which is based on the fact that the real numbers resulting from different measurements or settings never can be exact. In practice the values whose distance from a nominal value is less than an error or tolerance bound are identified as the nominal value. Thus two numbers (x_0 and x_1) can not be distinguished if their distance (δ_s) is not greater than the width of the tolerance interval (ε) (5). With other word x_0 and x_1 are ε indistinguishable if the inequality (5) is satisfied.

$$\delta_s(x_0, x_1) \le \varepsilon \tag{5}$$

The distance of two points can be measured with different unit of measurement e.g. cm and inch, which needs different ε values for the same result. Klawonn suggests the use of a scaling factor (function) for the avoidance of this problem in the distance measurement. Choosing a scaling factor whose value is not constant in the universe of discourse makes possible the measurement with different precision in different intervals depending on reliability or importance of the acquired data. The introduction of the scaling function means a transformation, which maps the original range of values to the interval [0,∞).

3.1 Distance in vague environment

The concept of vague environment [2] is characterized by its distance function δ_s. This distance is a transformed one, derived from the classical Euclidean distance and can be expressed by the formula (6). It can be considered a weighted distance whose weighting factor is the scaling function (s(x)).

$$\delta_s(x_1, x_2) = \left| \int_{x_1}^{x_2} \mathrm{s}(\mathrm{x})\mathrm{dx} \right| \tag{6}$$

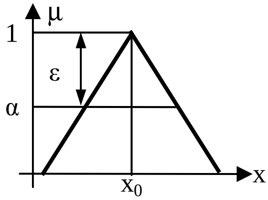

Fig. 2. Fuzzy set of points x which are ε-indistinguishable from x_0

The set of the points whose δ distance from a given point x_0 is not greater than ε can be considered as a fuzzy set. Its α-cuts contain the points which are ε=1-α indistinguishable from x_0.

From an other approach a fuzzy set can be described in the vague environment with a characteristic point (x_0) representing the core and the scaling function describing the shape of the set if the membership function is non-decreasing for values smaller than x_0 and it is non-increasing for values greater than x_0.

If it is possible to describe all the fuzzy partitions of the antecedent and consequent universes of the fuzzy rule-base, and the observation is a singleton, one can calculate the similarity measures of the antecedent fuzzy sets of the rule-base and the observation, and the similarity measures of the consequent fuzzy sets and the consequence (we are looking for) as vague distances of points.

3.2 The adequate scaling function

For generating a vague environment we have to find an appropriate scaling function, which describes the shapes of all the terms in the fuzzy partition [3]. There are three demands made on the scaling function. It should be integrable in the examined interval, it should give a good description of the shape of the fuzzy set and it should have low computational complexity. The description can be considered suitable only if it is possible to reconstruct the original fuzzy set from the characteristic point and the scaling function. The fuzzy partition can be easily described using the scaling function (3) suggested in [2] if the partition contains only one fuzzy set or the sets are not overlapping and each member function is piecewise linear, continuous and differentiable. A partition with two non overlapping sets and its scaling function are presented in Fig. 3.

$$s(x) = |\mu'(x)| = \left|\frac{d\mu}{dx}\right| \tag{3}$$

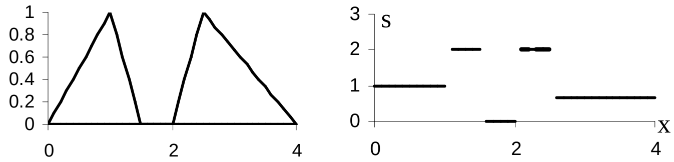

Fig. 3. Scaling function

The partition of a universe of discourse very often contains overlapping fuzzy sets. In this case the application of (3) is bound to the satisfaction of two conditions. The membership function should be triangle or trapezoid shaped and the partition should be a Ruspini one. In case of non fulfilment of the above mentioned conditions either a separate scaling function is needed in case of each fuzzy set for the description of the intervals containing overlapping membership functions or an approximate scaling function is needed, which can describe the overlapping sections of both neighbouring sets.

3.3 The approximate scaling function

The approximate scaling function is an approximation of the original scaling functions describing the fuzzy sets separately. The simplest way of generating this function is the linear interpolation (4) of the right side scaling factor of the left neighbouring term and the left side scaling factor of the right neighbouring term.

$$s(x)=\left\{\frac{s_{i+1}^{L}-s_{i}^{R}}{x_{i+1}-x_{i}}\cdot(x-x_{i})+s_{i}^{R}\ \middle|\ x\in[x_{i},x_{i+1}),\forall\, i\in[1,n-1]\right\} \tag{4}$$

where x_i - the core of the i^{th} set, s_i^L, s_i^R - the left and right side scaling factors of the i^{th} set, n - the number of the sets.

The drawback of the approximation (4) is that it can not handle the big differences between neighbouring scaling factors or crisp fuzzy sets correctly. In case of big differences, the bigger scaling factor "dominates" the smaller one. If one of the neighbouring fuzzy set is crisp (its scaling factor is infinite), the slope of the linearly interpolated scaling function is infinite too, so both the fuzzy sets described by this scaling function will be crisp. As a solution of this problem the adoption of a non-linear interpolative function is suggested in [3].

$$s(x)=\begin{cases}\dfrac{w_i}{(x_{i+1}-x_i+1)^{k\cdot w_i}-1}\cdot\left(\dfrac{(x_{i+1}-x_i+1)^{k\cdot w_i}}{(x-x_i+1)^{k\cdot w_i}}-1\right)+s_{i+1}^{L} & \middle|\ s_i^R\geq s_{i+1}^L\\[2ex] \dfrac{w_i}{(x_{i+1}-x_i+1)^{k\cdot w_i}-1}\cdot\left(\dfrac{(x_{i+1}-x_i+1)^{k\cdot w_i}}{(x_{i+1}-x+1)^{k\cdot w_i}}-1\right)+s_{i}^{R} & \middle|\ s_i^R< s_{i+1}^L\end{cases} \tag{5}$$

$$w_i=\left|s_{i+1}^{L}-s_{i}^{R}\right| \tag{6}$$

where $x\in[x_i,x_{i+1})$ $\forall$ $i\in[1,n-1]$, k - constant factor of sensitivity for neighbouring scaling factor differences.

The above function has same useful properties. If the neighbouring scaling factors are equals, s(x) is linear. If one of the neighbouring scaling factors (e.g. S_i^R) $s_i^R\to\infty$ and the other one is finite, in case of $x\in[x_i,x_{i+1})\Rightarrow$ s(x) is infinite in the neighbourhood of x_i and its value is null in any other point of the interval. Similarly if $s_{i+1}^L\to\infty$ and s_i^R is finite and $x\in[x_i,x_{i+1})\Rightarrow$ s(x) is infinite in the neighbourhood of x_{i+1} and its value is null in any other point of the interval.

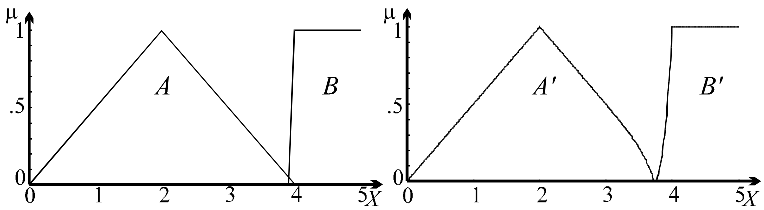

Fig. 4. Original and reconstructed fuzzy partition

Fig. 4. presents a fuzzy partition and its reconstructed version in case of the application of the scaling function (5). The value of 1 was used for the constant k.

If all the vague environments of the antecedent and consequent universes of the fuzzy rule base can be generated, the fuzzy rules can be characterised by points in the vague environment of the fuzzy rule base too. In this case the approximate fuzzy reasoning can be handled as a classical interpolation task. So any interpolation, extrapolation, or regression methods can be adapted very simply for approximate fuzzy reasoning [3].

4 Conclusions

Distance based similarity measures of fuzzy sets have a high importance in reasoning methods handling sparse fuzzy rule bases. The rule antecedents of the sparse fuzzy rule bases are not fully covering the input universe. Therefore the applied similarity measure has to be able to distinguish the similarity of non-overlapping fuzzy sets, too. The distance based similarity measures are such a measures. To give an overview of the distance based similarity measures of fuzzy sets, two of the main existing concepts are briefly introduced in this paper.

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