Johanyák, Z. C., Kovács S.: Single Rule Reasoning Methods in Fuzzy Rule Interpolation, Doktoranduszok Fóruma, Miskolci Egyetem, 9 November 2005, pp. 75-80., http://johanyak.hu

SINGLE RULE REASONING METHODS IN FUZZY RULE INTERPOLATION

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This paper intends to give a survey on some reasoning methods applicable in the second step of the generalized methodology of the fuzzy rule interpolation. First the problem of the approximate fuzzy reasoning and the generalized methodology as a possible solution as well as a framework is reviewed briefly followed by the presentation of the methods ST, FPL and SRM.

1. INTRODUCTION

Systems applying fuzzy logic rule bases produce the output as a result of a fuzzy reasoning process. Their rule base can be dense or sparse depending on whether a matching rule for all the possible input values exists, or not. In lack of a proper rule hit by the observation, the classical reasoning techniques like Zadeh's (CRI), Mamdani's, Larsen's, etc. cannot afford an acceptable output. This is why an approximate reasoning technique should be adopted when the rule base is sparse in order to ensure a proper result for all the possible observations.

There are several methods in the literature that can be applied in cases when the density condition is not fulfilled. Their largest family can be characterized by the feature that its members produce the result by rule interpolation taking into consideration two or more rules of the fuzzy rule base. These methods can be broken down into two groups depending on whether they are producing the estimated conclusion directly or they are interpolating an intermediate rule first.

Representative members of the first group are among others the KH method [1] proposed by Kóczy and Hirota, which is the first developed one, the MACI [2], the ARVE [3] introduced by Kovács and Kóczy, and the IMUL proposed by Wong, Gedeon and Tikk [4]. The structure of the methods belonging to the second group can be described best by the generalized methodology (GM) defined by Baranyi et al. [5]. Typical members of this group are e.g. the technique family proposed by Baranyi et al. in [5], the ST method [6] suggested by Yan, Mizumoto and Qiao and the IGRV [7] developed by Huang and Shen.

2. GENERALIZED METHODOLOGY

The GM was proposed by Baranyi, Kóczy and Gedeon in [5] for the task of the fuzzy rule interpolation. Reference points (RP), which can be identical with e.g. the centre points of the cores, are used for the characterization of the position of fuzzy sets. The distance of fuzzy sets is expressed by the Euclidian distance of their

reference points. The interpolation is broken down into two steps. In the first step an interpolated rule is produced, whose antecedent (A^i) has at least a partial overlapping with the observation (A^*) and whose RP coincides with RP(A^*). The solution of this task is divided into three stages. First by the help of a set interpolation technique A^i is produced. Next the reference point of the conclusion (B^i) is interpolated going out from the position of RP(A^*) and the reference points of the sets involved in the rules taken into consideration. Hereupon B^i is determined similarly to A^i.
The approximated rule is considered as a member of the extended rule base in the second step. The conclusion (B^*) corresponding to A^* is produced by the help of this rule. This step is discussed in detail in section 3. Owing to the modular structure of the methodology in both of the steps one can choose from several potential methods if some conventional elements (e.g. distance measure, reference point) are used consequently.

3. SINGLE RULE REASONING METHODS

The antecedent set (A^i) of the interpolated rule generally does not fit perfectly to the observation (A^*), therefore some kinds of special single rule reasoning techniques (SRRT) are needed in the second step. Further on three methods are reviewed, which are applicable for the mentioned task. For the sake of better lucidity the same notation is applied for presenting the individual techniques.

3.1. Similarity Transfer method

The ST method was introduced by Yan, Mizumoto and Qiao in [6]. Its key idea is that there is a common similarity in the antecedent and consequent parts. The technique is built up from two stages. First the similarity between A^* and A^i is measured. Next B^* is constructed from B^i according to the transferred similarity from the antecedent part.

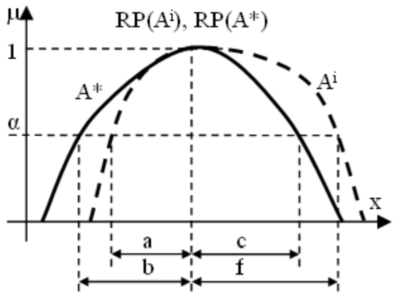

Figure 1.

The method is an α-cut based technique. A lower and an upper similarity value is defined for each α-cut. They can be formalized through (1) and (2) according to [6] and the notation structure of figure 1.

$$S_L\left(A^*, A^i, \alpha\right) = \frac{b}{a} = \frac{d\left(\inf\left\{A_\alpha^*\right\}, RP\left(A^*\right)\right)}{d\left(\inf\left\{A_\alpha^i\right\}, RP\left(A^i\right)\right)} \tag{1}$$

$$S_U\left(A^*, A^i, \alpha\right) = \frac{c}{f} = \frac{d\left(\sup\left\{A_\alpha^*\right\}, RP\left(A^*\right)\right)}{d\left(\sup\left\{A_\alpha^i\right\}, RP\left(A^i\right)\right)} \tag{2}$$

where S_L and S_U are the lower and upper similarity values, *inf* and *sup* are the lower and upper endpoints of the α-cut. B* is determined by its α-cuts conserving the lower and upper similarity ratio measured on the antecedent side ($S_{L/U}\left(B^*, B^i, \alpha\right) = S_{L/U}\left(A^*, A^i, \alpha\right)$).The method was worked out for the case of convex and normal fuzzy (CNF) sets. It is simple and it has low computational complexity.

3.2. Techniques based on the revision principle

The techniques suggested in [5] for the second step of the generalized methodology are based on the revision principle introduced by Shen, Ding and Mukaidono [8]. Prior to the detailed presentation of these techniques some definitions are necessary for their better understanding.

3.2.1. Definitions

The *interrelation function* (IR) is a mapping between the elements of two fuzzy sets. The involved sets can belong to the same partition (Fig. 2.a) or to two different universes (antecedent and consequent). The latter case usually occurs when the sets are bounded by a rule (Fig. 2.b). The IR defines which points are related to each other. In [5] some suggestions are made for its generation. According to them in case of convex and smooth shaped linguistic terms the endpoints of the support of the first set (e.g. A^i in Fig. 2.a) are mapped to the endpoints of the support of the second set (e.g. A* in Fig. 2.a) and the reference points are mapped together (e.g. $RP(A^i)$ and RP(A*) in Fig. 2.a) considering the IR piece-wise linear.

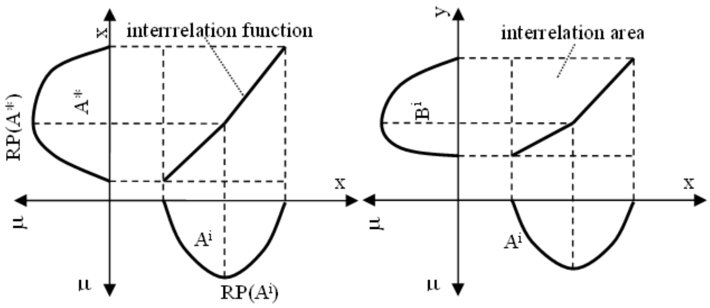

Figure 2.a. Figure 2.b.

In case of polygonal or non-convex shaped fuzzy linguistic terms first a set of characteristic points should be determined on both shapes followed by their linking together (e.g. Fig. 3 1st quarter). Some suggestions for the selection and mapping of characteristic points are made in [5] for this case, too. The rectangle defined by the endpoints of the supports of the sets is called the *interrelation area* (IRA).

The *semantic relation function* (SR) is a mapping between the membership values of the interrelated points of two fuzzy sets. Similar to IRA the rectangle containing the semantic relation curves is called the *semantic relation area* (SRA). The SR and IR for the same set pair are dependent on each other. Knowing one of them the other can be determined easily. In the example presented on figure 3 the SR plotted in the third quarter contains two different curves defining the semantic relation between the left (SR_l) and right (SR_r) flanks of the interrelated sets.

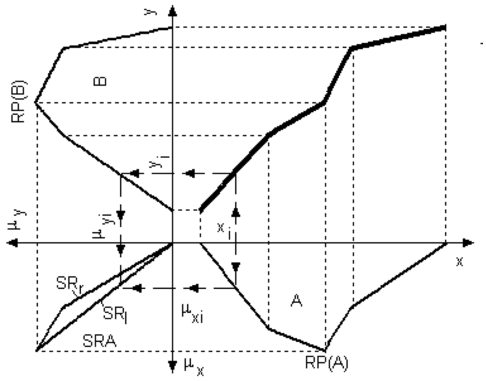

Figure 3.

Going out from the point x_i that belongs to the left edge of A the respective SR_l point can be determined following the dashed lines in the directions given by the arrows.

3.2.2. Transformations

As a precondition of the application of the techniques based on the revision principle (FPL and SRM) it should be mentioned that the support of the antecedent set has to coincide with the support of the observation. This is generally not fulfilled. In such cases the fuzzy relation (rule) obtained in the first step of the generalized methodology is transformed in order to meet this condition.
The technique called *Transformation of the Fuzzy Relation* (TFR) [5] transforms (stretches or shrinks) the interrelation area of the new rule proportionally by the help of set transformations in order to ensure the needed coincidence of the supports. The TFR transforms the antecedent (A^i) and consequent (B^i) sets separately, but in a similar way. Further on only the transformation of A^i is presented.
First an interrelation function is generated between A* and A^i in the usual way (Fig. 4.a). In case of A* only the position of the RP and the position of the endpoints of the support are relevant. Hereupon A^i is transformed obtaining A^t whose support coincides with the support of A*. The membership value of each point in A^t is equal to the membership value of its interrelated point in A^i. The transformed IR that gives the mapping between the points of A^t, B^t is constructed in the same manner as presented before. The presented technique conserves the piece-wise linearity, the position of the RP and the height of the original sets.

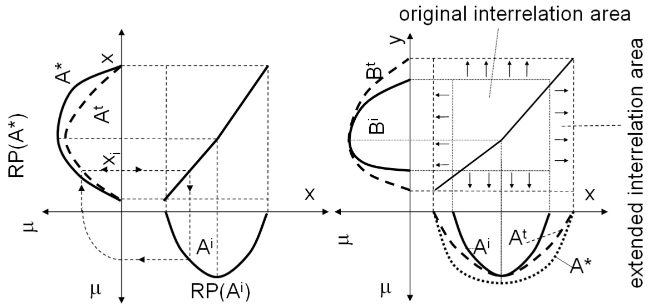

Figure 4.a. Figure 4.b.

The application of the technique SRM demands the equality of the heights of the rule antecedent (A^t) and the observation (A*) beside the above mentioned precondition. This can imply the need for the transformation of the SRA, too.

The algorithm called *Transformation of the Semantic Relation* modifies the SRA corresponding to the height of A*. The literature [5] suggests a normalization of the relation followed by a renormalization at the end of the SRM. In this paper a simplified solution is introduced. The sets belonging to the previously transformed relation (A^t, B^t) are transformed again into A^{st} and B^{st} in order to reach the needed equality. This transformation is described by the formulas (3) and (4):

$$\mu_{A^{st}}(x) = \mu_{A^t}(x) \cdot \frac{height(A^*)}{height(A^t)} \tag{3}$$

$$\mu_{B^{st}}(x) = \min\left(1, \mu_{B^t}(x) \cdot \frac{height(A^*)}{height(A^t)}\right) \tag{4}$$

During the set transformations the IRA remains unmodified. The new SR is determined from A^{st}, B^{st} and IRA in the well-known way (Fig. 3). Important features of the technique are that the change is continuous, it conserves the piece-wise linearity of the sets as well as it leaves the IRA unmodified. The aspect of the SR is conserved only if the condition $\mu_{B^t}(x) \cdot \frac{height(A^*)}{height(A^t)} \le 1$ is fulfilled.

3.2.3. Inference by Fixed Point Law

The FPL [8][5] goes out from the transformed sets A^t, B^t and the transformed IRA. First an IR is generated between A* and A^t. Next the difference between the membership values of each interrelated point pair is calculated. This deviation is used in the course of the determination of the approximated conclusion from the transformed consequent B^t taking into consideration the interrelation between A^t and B^t.

3.2.4. Inference by Semantic Revision based Method

The SRM goes out from the transformed sets A^{st}, B^{st} and the transformed relation areas (IRA and SRA). In the literature [8] two kinds of SRM techniques (I and II) are presented. Further on a simplified and unified version of them is discussed. It is supposed that between A* and B* there exists the same IR and SR as between A^{st} and B^{st}. It means that substituting A^{st} by A* and abandoning B^{st} the approximated conclusion can be determined using the existing IR and SR. The set B* is obtained in similar mode as presented in figure 3. The dashed path is the same, but now the starting point is y_i belonging to B*.

4. CONCLUSIONS

This paper was focused on a group of techniques which can be applied in the second step of the Generalized Methodology (GM) of the fuzzy rule interpolation. After an introductory section presenting the subject of rule interpolation in case of fuzzy systems disposing of a sparse rule base, the concept of the GM was overviewed briefly. Then three single rule inference techniques (ST, FPL and SRM) were studied more detailed. The ST technique has the smallest computational complexity, but its applicability is restricted to the convex and normal fuzzy (CNF) set case. The advantage of the FPL and SRM is their applicability in cases where the normality of the fuzzy sets cannot be satisfied.

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