Johanyák, Zs. Cs., Kovács, Sz.: Survey on three single rule reasoning methods,
A GAMF Közleményei, Kecskemét, XXI. évfolyam (2006-2007), ISSN 0230-6182, pp. 75-86
http://www.johanyak.hu

Survey on three single rule reasoning methods

Johanyák, Zsolt Csaba [1] – Kovács, Szilveszter [2]

Single Rule Reasoning (SRR) methods aim the determination of the conclusion from the observation and an intermediate (interpolated) rule. They are applied in fuzzy systems, which use an inference technique that follows the concepts of the Generalized Methodology (GM) [1] of Fuzzy Rule Interpolation (FRI).

This paper surveys and evaluates three SRR methods, namely SURE-p [2], SURE-LS [4] and REVE. The first two are overviewed briefly by recalling their main steps and essential features that are necessary for the evaluation and comparison. REVE is a new method based on the concept Vague Environment (VE). Therefore the paper contains its presentation in details.

1. Introduction

One reason of the wide popularity of Fuzzy Logic Systems in various application areas, such as control engineering, expert systems, pattern recognition, operation research, decision support systems etc. is inherited from the benefits of the rule based knowledge representation. In the most common Zadeh-Mamdani-Larsen Fuzzy Logic Reasoning method, where the original concept of Zadeh about linguistic terms [16] is applied for reasoning by Compositional Rule of Inference (CRI), as it is was proposed by Mamdani [11] and Larsen [10], the knowledge is represented by fuzzy rules.

Both the antecedents and the consequents of the Zadeh-Mamdani-Larsen type fuzzy rules are built up on fuzzy linguistic variables (fuzzy sets). The rules of the fuzzy rule base are interpreted as relations and the conclusion is formed as the composition of the observation fuzzy set and the fuzzy rule base relation.

Another popular fuzzy rule based reasoning method was introduced by Takagi and Sugeno in [12] and [13]. In their fuzzy rule representation the antecedents of the rules are similar to those that are used by Zadeh-Mamdani-Larsen method,

[1] Senior lecturer, Kecskemét College, GAMF Faculty, Sándor Kalmár Institute of Information Technology, Department of Information Technology
[2] PhD, associate professor, University of Miskolc, Department of Information Technology

but the conclusions are crisp functions of the input variables. In the Takagi-Sugeno type reasoning process the matching of the fuzzy observation and the fuzzy rule antecedents is interpreted in a similar way as in the CRI, but the conclusion (having crisp functions instead of fuzzy set rule consequences) is formed as a convex combination of the rule consequent functions weighted by the level of rule matching of the corresponding rules.

Both of the above mentioned classical inference methods determine the conclusion by means of rule-matching. They match the rule premises and the observation, and the conclusion is calculated as a weighted combination of rule consequents (fuzzy set consequences, or the crisp consequent function) with non-zero matching, where weights depend on the degree of matching.

Therefore, if the antecedent parts of the fuzzy rules are not covering sufficiently the observation space, may exists an observation that does not match to any of the rule antecedents, and hence the classical fuzzy rule inferences can not gain any conclusion. Such rule bases are called "sparse" fuzzy rule bases. They can arise from incomplete expert knowledge (lack of knowledge), inadequate parameter optimisation (system "tuning"), or as a result of a complexity reduction by eliminating rule redundancies. In any of the above mentioned cases of reasoning situation, an observation may appear from which the classical fuzzy inference cannot generate meaningful conclusion.

Such situations can be treated by non-classical fuzzy reasoning techniques, namely approximate fuzzy inference methods. Most of these methods are based on Fuzzy Rule Interpolation Techniques (FRIT). FRITs can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the current observation. Therefore the rule base of a system applying a FRIT is not necessarily complete; it should contain only the most significant fuzzy rules without risking the chance of having no conclusion for some of the observations.

Since 1991 numerous FRITs have been proposed. They can be divided into two main groups. The first group produces the approximated conclusion from the observation directly; therefore its members are called "one-step" methods. The members of the second group reach the target in two steps. In the first step they interpolate a new rule whose antecedent part overlaps the observation at least partially. The estimated conclusion is determined in the second reasoning step by a Single Rule Reasoning (SRR) method based on the similarity of the observation and the antecedent part of the newly interpolated rule. The "two-step" methods follow the concepts laid down by the Generalized Methodology (GM) of Fuzzy Rule Interpolation (FRI) introduced in [1] by Baranyi et al.

The rest of this paper is organised as follows. Section 2 defines a set of conditions for the evaluation and comparison of different SRR methods. Section

3 and 4 surveys briefly and evaluates the techniques SURE-p and SURE-LS. We introduce and evaluate a new revision method called REVE in section 5.

2. Conditions on revision methods

In order to facilitate the evaluation and comparison of the techniques being surveyed we have compiled a set of conditions based on the General Conditions on rule interpolation methods introduced in [5]. They are the followings.

1. *Avoidance of the abnormal conclusion*. The estimated fuzzy set should be a valid one. This condition can be described by the constraints (1) and (2) according to [14].

   $$\inf\{B^*_\alpha\} \le \sup\{B^*_\alpha\} \qquad \forall \alpha \in [0,1] \tag{1}$$

   $$\inf\{B^*_{\alpha 1}\} \le \inf\{B^*_{\alpha 2}\} \le \sup\{B^*_{\alpha 2}\} \le \sup\{B^*_{\alpha 1}\} \qquad \forall \alpha_1 < \alpha_2 \in [0,1] \tag{2}$$

   where *inf* and *sup* are the lower and upper endpoints of the current α-cut of the fuzzy set.

2. *Compatibility with the rule base*. This means the condition on the validity of the modus ponens, namely if an observation coincides with the antecedent part of a rule, the conclusion produced by the method should correspond to the consequent part of that rule.

3. *The fuzziness of the approximated result*. There are two opposite approaches in the literature related to this topic [15]. According to the first subcondition (3.a), the less uncertain the observation is the less fuzziness should have the approximated consequent. The second approach (3.b) originates the fuzziness of the estimated consequent from the nature of the fuzzy rule base. Thus, crisp conclusion can be expected only if all the consequents of the rules taken into consideration during the interpolation are singleton shaped, i.e. the knowledge base produces certain information from fuzzy input data.

4. *Conserving the piece-wise linearity*. If the fuzzy sets of the consequent partitions including here also the consequents of the intermediate rule are piece-wise linear, the approximated sets should conserve this feature.

5. *Applicability in case of multidimensional antecedent universe*.

6. *Shape invariance.* The method should be applicable for all kinds of linguistic term shape types.

3. SURE-p

The method SURE-p (Single rUle REasoning based on polar cuts) introduced in [2] determines the shape of the conclusion from its polar cuts. The concept of polar cuts originally introduced in [3] is based on the application of a polar coordinate system, whose origin is placed in the reference point of the linguistic term.

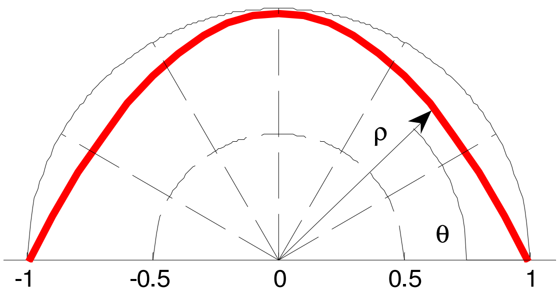

Figure 1. Polar cut

A polar cut is defined by a value pair $\{\rho,\theta\}$ where ρ is the polar distance of the point situated on the shape of the set at the polar angle θ (fig. 1). Similar to the case of α-cuts an extension and a resolution principle can be defined for polar cuts as well. The extension principle states that the solution of a problem regarding a fuzzy set can be found by solving it first for its polar cuts and then extending the results to the fuzzy case. The resolution principle expresses that a convex fuzzy set can be decomposed to polar cuts.

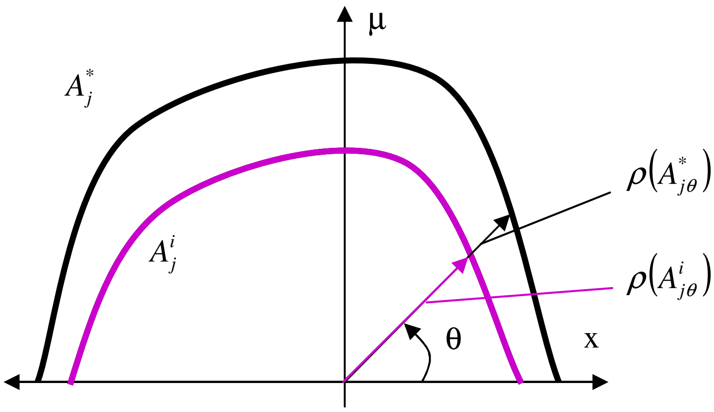

Figure 2. Difference between the polar distance of the antecedent set and the polar distance of the observation at the polar level θ

The further calculations can be simplified if the range of all the antecedent and consequent partitions is normalized to the unit interval ([0,1]). Therefore in the following this case is going to be presented. Most of the calculations of SURE-p can be done for each output dimension separately, therefore they can be made parallel if the computing environment it enables.

The basic idea of the method is the conservation of the average difference measured on the antecedent side. Therefore for each input dimension and for each polar level one calculates the differences between the polar lengths of the observation and the rule antecedent (fig. 2). Next for each polar cut the average distance is determined.

The conclusion results from the revision of the consequent of the new rule by the average difference at the corresponding θ level. The abnormal results (e.g. fig. 3) are avoided by a verification and correction algorithm, which is part of the method as well.

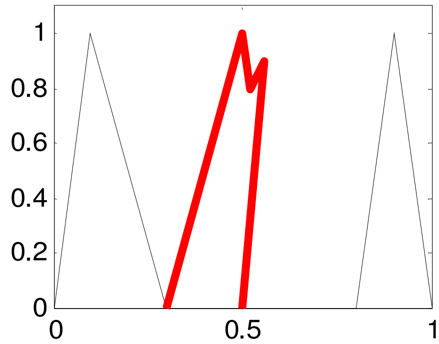

Figure 3. Abnormal conclusion

In case of a perfect overlapping between the observation and the antecedent of the interpolated rule the conclusion will be the same as the consequent of the rule. The method does not conserve the piece-wise linearity but it is well applicable in case of multidimensional antecedent universes. SURE-p is shape invariant; its main advantage is that it can handle subnormal linguistic terms as well. The method fulfils conditions 1,2,3.a, 5 and 6.

4. SURE-LS

The revision method SURE-LS (Single rUle REasoning based on the method of Least Squares)[4] was developed especially for the case when all linguistic terms of a consequent partition belong to the same shape type (e.g. singleton, triangle, trapezoid, etc.) and all characteristic (break) points are situated at the same α-levels. It also means that the height of all sets is the same.

In such circumstances it seems to be a natural demand that the conclusion should also adhere to this regularity. Thus one does not seek an arbitrary shaped conclusion but a special form with the characteristic points having predetermined ordinate values. Therefore only the abscissas of the characteristic points have to be calculated.

SURE-LS applies an α-cut based approach for this task. It uses a set of α-levels compiled together by taking into consideration the break-point levels of all antecedent dimensions and the current consequent partition. The calculations are done separately for the left and right flanks. On each side for each level it calculates the weighted average of the distances between the endpoints of the α-cuts of the rule antecedent and the observation set. The weighting makes possible to take into consideration the different antecedent dimensions (input state variables) with different influence.

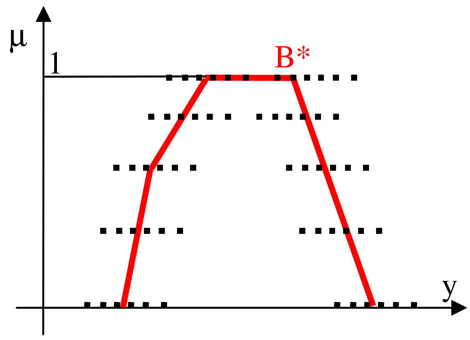

Figure 4. The shape of the conclusion calculated by SURE-LS

The basic idea of the method is slightly similar to the concept of SURE-p, namely the conservation of the weighted average differences measured on the antecedent side. Here the differences are measured in horizontal direction and the revision results in an intermediate set or an array of points. The conclusion with the desired shape type is calculated from these applying the method of Least Squares (fig. 4).

Due to the last step of the calculation the resulting set is always a valid one and it conserves the piece-wise linearity. Similar to the previous method if the rule antecedent fits the observation perfectly the conclusion will be identical with the consequent of the rule. The method is developed for systems with a multidimensional antecedent universe. It does not met the shape invariance criteria because it is applicable only in the case when all the linguistic terms of the partition can be described by a single shape type. The main advantage of the method is its low computational complexity. It fulfils the conditions 1, 2, 3.a, 4 and 5.

5. REVE

In [7] Klawonn introduced a new approach to handle uncertainty or imprecision concentrating on the idea of indistinguishability of values whose difference is small. The new concept was called Vague Environment (VE) and it was enhanced and applied for the development of a fuzzy rule interpolation technique working with singleton type observations (FIVE) by Kovács in [8]. Later the method was extended to the case of fuzzy input values ([9]) as well.

REVE (Revision mEthod based on the Vague Environment) is a novel single rule reasoning method developed by the authors of this paper. It is also based on the concept Vague Environment and it is suggested as a complementary of the fuzzy set interpolation method VESI [6]. However, it can be also applied as single rule reasoning technique in case of any two-step fuzzy rule interpolation method that follows the concepts of GM [1].

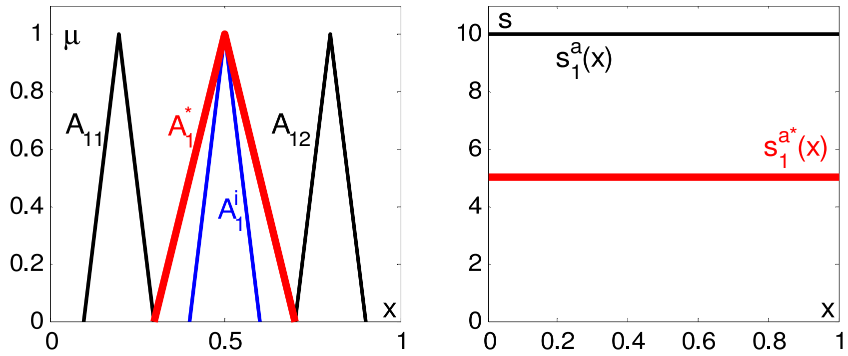

Figure 5. One dimensional antecedent universe of discourse, observation (A_1^* - bold line), interpolated antecedent set (A_1^i) and the corresponding scaling functions

The main idea of REVE is the conservation of the scaling function ratio in single rule reasoning. Having VEs (and hence scaling functions) on both the rule antecedent and the consequent sides, the scaling function ratio between the rule antecedent and the observation should be equal to the scaling function ratio between the rule consequent and the demanded conclusion. In other words, the similarity of fuzzy sets is expressed in the form of the similarities of the corresponding VEs, in their scaling function ratio. For multidimensional antecedent universes, the basic "scaling function ratio" idea could be simply extended to "mean scaling function ratio" too.

The main steps of the proposed REVE method are introduced in the followings. In the first step of the proposed method all input and output partitions are normalized to the unit interval. Next REVE calculates for each input dimension the ratio of the scaling function describing the VE of the observation ($s_i^{a*}(x)$, see fig. 5) and the scaling function of the antecedent partition ($s_i^a(x)$)

$$r_i^a(x) = \frac{s_i^{a*}(x)}{s_i^a(x)}, \tag{3}$$

where i is the number of the current antecedent dimension. Hereupon the harmonic mean of the antecedent ratios is calculated

$$mr^a(x) = \frac{n_a}{\sum_{i=1}^{n_a} \frac{1}{r_i^a(x)}}, \tag{4}$$

where n_a is the number of antecedent dimensions. The rest of the calculations are done separately for each consequent dimension. Further on the case of the j^{th} output dimension is considered.

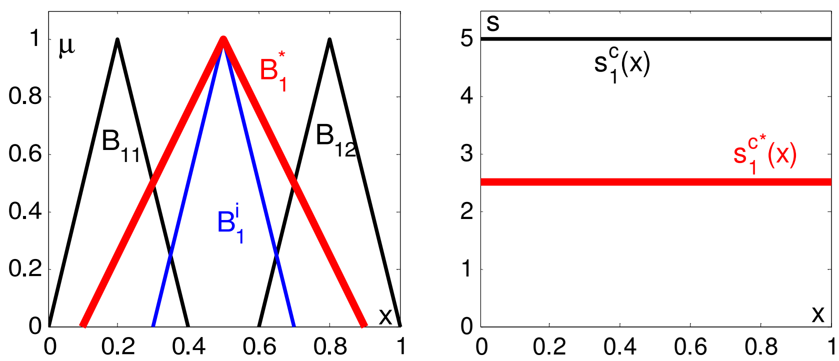

Figure 6. One dimensional consequent universe, interpolated consequent set (B_1^i), conclusion (B_1^*-bold line), and the corresponding scaling functions

The basic idea of REVE is the principle of the conservation of the mean scaling function ratio. Thus the scaling function inducing the Vague Environment of the conclusion is determined by considering the same ratio between the scaling functions of the conclusion and the consequent partition as the mean scaling function ratio calculated on the antecedent side

$$r_j^c(x) = mr^a(x) = \frac{s_j^{c*}(x)}{s_j^c(x)}, \tag{5}$$

where $s_j^c(x)$ is the scaling function in the j^{th} consequent dimension, and $s_j^{c*}(x)$ is the scaling function describing the conclusion in the j^{th} dimension (see fig. 6). The scaling function of the conclusion results from the formula (5) as follows

$$s_j^{c*}(x) = s_j^c(x) \cdot mr^a(x). \tag{6}$$

Figure 5 and 6 illustrates a simple numerical example using a SISO (Single Input Single Output) system and applying on the consequent side the same ratio equal to 0.5 that was measured on the antecedent side.

REVE has low computational complexity and its effectiveness increases with the number of consequent dimensions. It is due to the fact that the calculations on the antecedent side have to be done only once and the conclusion sets are determined separately in each consequent dimension. Thus several steps can made parallel if the computing environment it enables. The computational needs can be reduced further when the method is applied together with VESI by using the same pregenerated antecedent and consequent VEs.

The application of the harmonic mean ensures that the conclusion will be crisp ($s_j^{c*}(x) = \infty$) if and only if all observation sets are crisp ($s_i^{a*}(x) = \infty\ \forall i$). The condition on compatibility with the rule base is also fulfilled (if the scaling functions of the observation and the rule antecedent are the same in all dimensions the conclusion will be equal to the rule consequent). Due to the ratio based conservation principle the changing direction of the fuzziness of the conclusion follows the changing direction of the fuzziness of the observation. Owing to these two features is condition 3.a fulfilled.

Another advantage of the proposed REVE method is the lack of verification and correction steps required in many other methods for gaining convex valid fuzzy conclusion, hence the conclusion of REVE is always a convex valid fuzzy set.

The above mentioned benefits are the pledge of the real-time applicability of the proposed VESI-REVE method pair, as in real-time systems the quick response (the speed of inference) is an essential requirement

The main disadvantage of the proposed method is the lack of compatibility with the rule base in case, when only an approximate scaling function can be determined in any of the antecedent or consequent partitions. In addition the piece-wise linearity is also not always conserved. Hence the method fulfils the conditions 1, 3.a, 5 and 6 only.

6. Conclusions

Fuzzy Rule Interpolation based inference techniques ensure an acceptable output for fuzzy systems applying either full covering (dense), or sparse rule bases. A significant group of these techniques follows the two-step approach by interpolating first a new rule in the position of the observation and then calculating the conclusion by revising the rule consequent sets.

Table 1. Feature matrix of single rule reasoning methods

	1	2	3.a	4	5	6
SURE-p	✓	✓	✓		✓	✓
SURE-LS	✓	✓	✓	✓	✓	
REVE	✓		✓		✓	✓

This paper surveyed and evaluated three SRR methods, the last of them being a newly introduced one. The results of the evaluation are summarized in table 1. None of the presented techniques fulfilled all the criteria, but all of them had their advantages that justify their application for the solution of one or more problem types. The evaluation facilitates the selection of the proper method as well.

ACKNOWLWDGEMENT

This research was supported by Kecskemét College GAMF Faculty grant no: 1N076/2006.

REFERENCES

[1] Baranyi, P., Kóczy, L. T. and Gedeon, T. D.: A Generalized Concept for Fuzzy Rule Interpolation. IEEE Trans. on Fuzzy Systems, vol. 12, No. 6, 2004, pp 820-837

[2] Johanyák, Zs. Cs., Kovács Sz.: Fuzzy Rule Interpolation Based on Polar Cuts, Computational Intelligence, Theory and Applications, Springer Berlin Heidelberg, 2006, ISBN 978-3-540-34780-4, pp. 499-511

[3] Johanyák, Zs. Cs., Kovács Sz.: Fuzzy set approximation using polar co-ordinates and linguistic term shifting, SAMI 2006, 4th Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence, Herl'any, Slovakia, January 20-21 2006, ISBN 963 7154 44 2, pp. 219-227

[4] Johanyák, Zs. Cs., Kovács, Sz.: Fuzzy Rule Interpolation by the Least Squares Method, 7th International Symposium of Hungarian Researchers on Computational Intelligence (HUCI 2006), November 24-25, 2006 Budapest, ISBN 963 7154 54 X, pp. 495-506

[5] Johanyák, Zs. Cs., Kovács Sz.: Survey on various interpolation based fuzzy reasoning methods, Production Systems and Information Engineering Volume 3 (2006), HU ISSN 1785-1270, pp. 39-56

[6] Johanyák, Zs. Cs.: Vague Environment Based Set Interpolation, A GAMF Közleményei, Kecskemét, XX. évfolyam (2006) (submitted for publication).

[7] Klawonn, F.: Fuzzy Sets and Vague Environments, Fuzzy Sets and Systems, 66, pp. 207-221, 1994.

[8] Kovács, Sz., Kóczy, L. T.:Application of an approximate fuzzy logic controller in an AGV steering system, path tracking and collision avoidance strategy, Fuzzy Set Theory and Applications , In Tatra Mountains Mathematical Publications, Mathematical Institute Slovak Academy of Sciences, vol.16, Bratislava, Slovakia, 1999, pp. 456-467

[9] Kovács, Sz.: Extending the Fuzzy Rule Interpolation "FIVE" by Fuzzy Observation, Theory and Applications, Springer Berlin Heidelberg, 2006, ISBN 978-3-540-34780-4, pp. 485-497

[10] Larsen, P. M.: Industrial application of fuzzy logic control. Int. J. of Man Machine Studies, 12(4):3-10, 1980.

[11] Mamdani, E. H. and Assilian, S.: An experiment in linguistic synthesis with a fuzzy logic controller. Int. J. of Man Machine Studies, 7:1–13, 1975.

[12] Sugeno, M.: An introductory survey of fuzzy control. Information Science, 36:59–83, 1985.

[13] Takagi, T. and Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. on SMC, 15:116–132, 1985.

[14] Tikk, D., Baranyi, P.: Comprehensive analysis of a new fuzzy rule interpolation method, IEEE Trans Fuzzy Syst., vol. 8, June 2000, pp. 281-296

[15] Tikk, D.: Investigation of fuzzy rule interpolation techniques and the universal approximation property of fuzzy controllers, Ph. D. dissertation, TU Budapest, Budapest, 1999.

[16] Zadeh, L. A.: Outline of a new approach to the analysis of complex systems and decision processes. IEEE Trans. on SMC, 3:28–44, 1973.

Három egyszabályos következtetési módszer áttekintése

Johanyák Zsolt Csaba - Kovács Szilveszter

Összefoglalás

Az egyszabályos következtetési módszerek célja a következmény meghatározása a megfigyelés és a köztes (interpolált) szabály ismeretében. Alkalmazásukra olyan fuzzy rendszereknél kerül sor, amelyek a fuzzy szabályinterpoláció általános módszertanát [1] követik.

Cikkünkben három egyszabályos következtetési eljárással foglalkozunk. Ezek a SURE-p [2], a SURE-LS [4] és a REVE. Egy értékelési és összehasonlítási követelményrendszer felállítását követően röviden ismertetjük a korábban már publikált első két módszert kiemelve az értékeléshez és összehasonlításhoz szükséges fő lépéseket és lényeges tulajdonságokat. Ezután bemutatjuk a REVE eljárást, ami egy a bizonytalan környezet [7][8] fogalmára épülő új módszer. Az eljárás menetét részletesen ismertetjük.

Untersuchung von drei Einzelregel-basierten Inferenzmethoden

Johanyák, Zsolt Csaba – Kovács, Szilveszter

Zusammenfassung

Einzelregel-basierten Inferenzmethoden sind entwickelt für die Rechnung der Folge ausgehend von der Beobachtung und der interpolierten Regel. Sie sind benutzt bei solchen Fuzzy Systemen, die der Generalisierten Methodik für Fuzzy Regel-Interpolation [1] folgen.

Dieser Artikel untersucht die Einzelregel-basierte Inferenzmethoden SURE-p [2], SURE-LS [4] und REVE. Nach der Zusammenstellung einem Erforderungssystem für Bewertung und Vergleich werden die ersten zwei Methoden, die schon früher publiziert wurden, kurz präsentiert. Die wichtigste Schritte und wesentliche Eigenschaften, die für Bewertung und Vergleich notwendig sind, werden hervorgehoben. Danach wird ein neues Verfahren eingeführt, das auf dem Konzept Vage Umgebung [7][8] basiert ist. Die Methode REVE wird ausführlich dargelegt.