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Student Evaluation Based On Fuzzy Rule Interpolation

Zsolt Csaba Johanyák

Kecskemét College, Institute of Information Technologies, Kecskemét, Hungary,
H-6000 Kecskemét, Izsáki út 10.

johanyak.csaba@gamf.kefo.hu

Abstract

The evaluation of narrative student responses contains always vague elements that originate from the fact that the students performance cannot be clearly ranked in one or another category, which makes this area a natural application field for fuzzy set theory based solutions. This paper presents a new evaluation method that allows taking into consideration three main aspects, i.e. the accuracy of the response, the correctness of the technical terms' use, and the total time necessary for answering the questions. The evaluator can express the uncertainty in his or her judgment by fuzzy numbers. The method determines the final evaluation using fuzzy arithmetic and inference based on rule interpolation.

Keywords: *fuzzy student evaluation, SEFRI, fuzzy rule interpolation*

Mathematics Subject Classification: 68T37

CCS: I.2.1

1 INTRODUCTION

The evaluation of students' answerscripts, presentations, etc. contains a lot of steps that cannot be fully and precisely defined and therefore it neither can be fully automated. Also here belong the cases when technically it would be possible the automation, but the in-depth definition of the scoring rules would require enormous computational or time resources, which are not affordable. Thus the scoring usually is made by human evaluators. Their decisions always can contain subjective elements that can be tracked back to the person (different evaluators give different scores), to the time of the evaluation (the same evaluator assigns different scores for the same answer), and the rigidity of the applied scoring system (the evaluator has to express his or her opinion with one crisp number although there is a vagueness in it). Beside the subjective elements of the evaluation process in some cases another problem arises from the demand on establishing a clear ranking among the students, i.e. one should avoid the appearance of two or more identical scores.

In the last fifteen years several fuzzy set theory based evaluation supporting methods have been published. They offer solutions for one or more of the above described problems. Biswas (Biswas, 1994) proposed a particular (FEM) and a generalized (GFEM) method that were based on the vector representation of fuzzy membership functions and a special aggregation of the grades assigned to each question of the student's answerscripts. Chen and Lee (Chen and Lee, 1999) suggested a simple (CL) and a generalized (CLG) method that produced improvements by applying a finer resolution of the scoring interval and by including the possibility of weighting the four evaluation criteria. Nolan introduced (Nolan, 1998) a fuzzy classification model for supporting the grading of student writing samples in order to speed up and made more consistent the evaluation. Wang and Chen (Wang and Chen 2006) extended the CL/CLG method pair by introducing the evaluator's optimism as a new aspect, and by using type-2 fuzzy numbers for the definition of the satisfaction. Bai and Chen (Bai and Chen 2008) developed a method for the ranking of students that obtained the same total score during the traditional evaluation. They used a three-level fuzzy reasoning process. All of the above mentioned methods have their advantages and disadvantages; however, they cannot cover all the potential application areas.

In this paper, a new fuzzy set theory based student evaluation method called SEFRI is introduced. Its novelty can be summarized essentially in three main points: (1) it takes into consideration a new aspect, namely the correct use of the technical terms that was particularly demanded by our institution; (2) it applies fuzzy arithmetic for the calculation of the average score in case of two aspects; (3) it determines the final grade using fuzzy rule interpolation based inference. The rest of the paper is organized as follows. Section 2 gives a brief survey on the definitions, concepts, and methods necessary for the rest of the paper. Section 3 presents the SEFRI method.

2 PRELIMINARIES

2.1 Sum and Multiplication of Fuzzy Numbers

In this section we will recall some basic definitions and concepts regarding the fuzzy numbers and fuzzy arithmetic that will be necessary later in course of the student performance evaluation.

Definition 1 A fuzzy set *A* ($\mu_A : \Re \to [0,1]$) is called a fuzzy number if satisfies the following conditions (Fodor and Bede 2006).

1. *A* is normal, i.e. $\exists\, x \in \Re$ such that $\mu_A(x) = 1$, where μ_A is the membership function of *A*.
2. *A* is convex, i.e. $\mu_A(\lambda \cdot x + (1-\lambda)\cdot y) \geq min\{\, \mu_A(x), \mu_A(y)\}$, $\forall\, x, y \in \Re$, $\forall\, \lambda \in [0,1]$.
3. The membership function μ_A is at least piecewise continuous.
4. The set *A* is compact in $\Re$, i.e. $supp(A)$ is a bounded interval, where $supp(A) = \{\, x \in \Re \mid \mu_A(x) > 0\}$

Definition 2 The α-cut of a fuzzy set *A* is a bounded subset of the universe of discourse (here $\Re$) defined by

$$[A]^{\alpha} = \{\, x \in \Re \mid \mu_A(x) \geq \alpha;\ \alpha \in [0,1]\,\}. \tag{1}$$

Let us denote the by

$$[A]^{\alpha} = \left[\underline{a}^{\alpha}, \overline{a}^{\alpha}\right], \tag{2}$$

where $\underline{a}^{\alpha}$ and $\overline{a}^{\alpha}$ are the lower respective upper bounds of the α-cut, and

$$\underline{a}^{\alpha} = inf\left\{\, [A]^{\alpha}\, \right\}, \tag{3}$$

$$\overline{a}^{\alpha} = sup\left\{\, [A]^{\alpha}\, \right\}. \tag{4}$$

Definition 3 The sum of two fuzzy numbers is defined conform Zadeh's extension principle based on the α-cuts of the numbers by

$$[A+B]^{\alpha} = [A]^{\alpha} + [B]^{\alpha} = \left[\underline{a}^{\alpha} + \underline{b}^{\alpha}, \overline{a}^{\alpha} + \overline{b}^{\alpha}\right], \tag{5}$$

for every $\alpha \in [0, 1]$.

Definition 4 The scalar multiplication of a fuzzy number is defined conform Zadeh's extension principle based on the α-cuts of the numbers by

$$[\lambda \cdot A]^\alpha = \lambda \cdot [A]^\alpha = \begin{cases} \left[\lambda \cdot \underline{a}^\alpha, \lambda \cdot \overline{a}^\alpha\right] & if\ \lambda \geq 0, \\ \left[\lambda \cdot \overline{a}^\alpha, \lambda \cdot \underline{a}^\alpha\right] & if\ \lambda < 0, \end{cases} \tag{6}$$

for every $\alpha \in [0, 1]$.

We denote by $\Re_F$ the set of all fuzzy numbers.

2.2 Sparse Fuzzy Rule Base

A fuzzy rule base is the knowledge storage element of a system that applies fuzzy inference. In case of a multiple input multiple output (MIMO) system the knowledge is represented by fuzzy rules of form

$$\begin{aligned} &IF\ a_1 = A_{1x}\ AND\ a_2 = A_{2x}\ AND\ ...\ AND\ a_i = A_{ix}\ AND\ ...\ AND\ a_n = A_{nx} \\ &THEN\ b_1 = B_{1x}\ AND\ b_2 = B_{2x}\ AND\ ...\ AND\ b_j = B_{jx}\ AND\ ...\ AND\ b_m = B_{mx}, \end{aligned} \tag{7}$$

where n is the number of the input (antecedent) dimensions, a_i ($1 \leq i \leq n$) is the linguistic variable of the *i*th antecedent dimension, A_{ix} is one of the linguistic values of the *i*th antecedent dimension, m is the number of the output (consequent) dimensions, b_j ($1 \leq j \leq m$) is the linguistic variable of the *j*th consequent dimension, B_{jx} is one of the linguistic values of the *j*th consequent dimension.

Depending on the coverage of the antecedent space the rule base can be characterized as dense or sparse. In order to examine the coverage let us define first the activation degree of a rule.

Definition 5 The activation degree of a rule is

$$d = min\left\{ t\left(A_{1x}, A_1^*\right), t\left(A_{2x}, A_2^*\right), ..., t\left(A_{nx}, A_n^*\right) \right\}, \tag{8}$$

where A_i^* denotes the observation in the *i*th antecedent dimension and *t* is an arbitrary t-norm.

Definition 6 The activation degree of a rule base is determined by the highest rule activation degree as

$$dr = max\{ d_1, d_2, ..., d_r \}, \tag{9}$$

where *r* is the number of the rules.

Definition 7 A rule base is dense when $\forall A^* \in X, dr > 0$, where X denotes the multidimensional universe of discourse and A^* is the observation. In any other cases the rule base is considered sparse.

A dense rule base contains for each possible fuzzyfied input (observation) value at least one rule whose antecedent part intersects or overlaps the observation with a membership value greater than zero. Figures 1 and 2 illustrate a dense and a sparse fuzzy rule base in case of a two dimensional (A_1 and A_2) antecedent universe of discourse.

A sparse rule base has a lower storage demand than a dense one due to the reduced number of rules. Its application can speed up the system owing to the reduced complexity and time need for searching the rules that have an activation degree greater than zero. If the sparse rule base contains all the relevant rules the system based on it can be as precise as it would be in case of a dense rule base (Tikk et al. 2003). Besides, the knowledge representation is more transparent. Its advantages open a wide area of practical applications not only on field of expert systems but also on area of fuzzy control (Precup et al. 2008).

2.3 Rule Interpolation Based Fuzzy Inference

Theoretically in case of a dense rule base any fuzzy inference method can be applied. However, in case of a sparse rule base one has to use an approximate reasoning technique, which is usually based on fuzzy rule interpolation (FRI). Since the publication of the first FRI method by Kóczy and Hirota in (Kóczy and Hirota 1993) this field has become the subject of an intense research work. The FRI methods form two main groups.

The members of the first one calculate the output straight from the input and the two or more closest rules. For example Kóczy and Hirota's method, the vague environment based FIVE (Kovács 2006), and the coordinate transformation based MACI (Tikk and Baranyi 2000) follow this approach. The methods belonging to the second group interpolate first a new rule and then calculate the conclusion by its help. For example the generalized methodology of fuzzy rule interpolation (Baranyi et al. 2004), the least squares approximation based LESFRI (Johanyák and Kovács 2006), and Chen and Ko's transformation based technique (Chen and Ko 2008) apply this concept. Later on we will use LESFRI for fuzzy inference. Its detailed presentation is published in (Johanyák and Kovács 2006); therefore in this paper we will recall only its basic concepts.

The least squares based fuzzy rule interpolation (LESFRI) infers the conclusion in two steps. In its first step it interpolates a new rule, which involves three main tasks: the calculation of the antecedent sets' membership functions, the determination of the consequent sets' position, and the calculation of the consequent sets' membership functions. The first and the third task is solved by the FEAT-LS set interpolation method, while the position of the consequent stets is determined with an adapted version of the Shepard's crisp interpolation (Shepard 1968).

FEAT-LS determines the shape of the interpolated fuzzy set in two steps. First, it shifts all the existent sets in the current dimension of the universe of discourse in order to reach the coincidence between their reference points and the point of the interpolation. Figures 3 and 4 illustrate the situation before and after shifting. Then the method calculates the characteristic points (parameters) of the new linguistic value α-cut wise separately for the left and right flanks. In case of piece-wise linear membership functions only the α-levels corresponding to the breakpoints are necessary for the calculations. In case of a given flank and α-level the abscissa of the new point on the shape of the interpolated set is calculated as a weighted average of the corresponding points of the overlapped sets. The key idea of the weighting is that the sets situated originally in closer neighborhood of the interpolation point should have greater influence than others. One calculates the final shape from the approximated points taking into consideration the following two conditions: the result should be a valid fuzzy set, and the result should conform to the characteristic shape type of the partition (e.g. singleton, triangle, trapezoid, etc.). In order to fulfill them FEAT-LS calculates the membership function using the least squares method.

Supposing the existence of n input dimensions each rule can be represented in case of each output dimension by a point in the *n+1* dimensional rule space, which point is defined by the reference points of the antecedent sets and the reference point of the consequent set. Assuming the existence of regularity between these points LESFRI determines the reference point of the interpolated rule's consequent set extending Shepard's crisp interpolation. Thus one calculates the new point as a weighted average of the existent points. The weighting depends on distance between rule antecedents, which is defined as the Euclidean distance between the point representing the antecedent part of the interpolated rule in the antecedent space and the points representing the existing rules in the same space. The concept of the weighting is similar to the one used in the case of the set interpolation.

LESFRI generates the conclusion in its second step using the single rule reasoning method SURE-LS. Single rule reasoning takes into consideration only one rule whose antecedent sets overlap the observation in each dimension. In our case this condition is fulfilled by the first step of LESFRI. The key idea of the method is the modification of the rule consequent dependent on the dissimilarities between the antecedent and observation sets. The dissimilarities are measured α-cut wise by calculating the distances between the lower (upper) endpoints of the α-cuts in each antecedent dimension. Then an average difference is calculated from the differences measured in the individual dimensions and this value is used for the modification of the lower (upper) endpoint of the related α-cut of the interpolated rule's consequent set.

In order to preserve the characteristic shape type of the consequent partition one calculates the final membership function by the method of least squares. The left and right flanks are calculated separately. The number of necessary α-cuts depends on the membership function types used in the different dimensions. For example in the most simple case of triangle shaped normal fuzzy sets in all dimensions only two α-cuts at the levels 0 and 1 should be enough.

2.4 Traditional Evaluation of the Students' Performance

Although there is no standardized scoring guide in our institute usually the rating of assignments with narrative responses happens as follows. The total number of marks for an assignment or group of consecutive assignments is 100. This number is divided between the questions of the assignment(s).

Our institute does not use an explicit weight number set; the significance of a question is expressed by the number of marks a student can achieve in case of a perfect response.

The assignment of the actual number of marks is based on the expertise of the evaluator. At the end we calculate a total score calculated by summarizing the individual scores achieved in case of each question, and the final score is mapped to a five-graded scale. The grades are "unsatisfactory", "satisfactory", "average", "good", and "excellent". The mapping is standardized; the score intervals corresponding to the grades are presented in Table 1. They also can be described by the crisp sets on Figure 5.

3 A NEW APPROACH FOR STUDENTS' PERFORMANCE EVALUATION

In this section a new approach for student evaluation is presented, which is based on fuzzy arithmetic and inference. The method Student Evaluation based on Fuzzy Rule Interpolation (SEFRI) takes into consideration three aspects in course of the scoring, namely

- the accuracy of the response,
- the time necessary for answering the questions,
- the correct use of the technical terms.

The accuracy of the response is measured by the evaluator by the means of fuzzy numbers defined on the unit interval ($\mu_A(x):[0,1]\to[0,1]$). Theoretically each kind of membership function that fulfills the previously presented conditions on fuzzy numbers can be used for this task; however, the application of membership function types that can be described with a low number of parameters (e.g. triangle shaped or trapezoidal) is advisable.

Let us denote by *AC* the vector containing the accuracy values assigned to the responses of the current student

$$AC = [ac_1 \quad ac_2 \quad \ldots \quad ac_n], \tag{10}$$

where *n* is the number of the questions, and $ac_i \in \Re_F \quad (1 \le i \le n)$ is the fuzzy number describing the opinion of the evaluator regarding the response to the question Q_i.

The time need of the response (*t*) is a crisp number. Contrary to other approaches we do not measure the time need of the individually responses for each question, only the total amount of time necessary for the test is measured. It is because at our institution only the total time is prescribed. The students can partition it conform their needs. Further on we will work with the relative value of *t* reported to the total allowed response time (t_T)

$$t_r = \frac{t}{t_T}, \tag{11}$$

where $t_r \in [0,1]$.

The correct use of the technical terms is measured by the evaluator by the means of fuzzy numbers defined on the unit interval ($\mu_A(x):[0,1]\to[0,1]$). The considerations regarding the membership function type selection presented in case of the accuracy of the response also here are valid. We denote by *CU* the vector containing the values assigned to the responses of the current student

$$CU = [cu_1 \quad cu_2 \quad \ldots \quad cu_n], \tag{12}$$

where *n* is the number of the questions, $cu_i \in \Re_F \quad (1 \le i \le n)$ is the fuzzy number describing the opinion of the evaluator regarding the response to the question Q_i.

Let *SC* be the score matrix storing the maximal score values assigned to the individual questions

$$SC = [sc_1 \quad sc_2 \quad \ldots \quad sc_n], \tag{13}$$

where *n* is the number of the questions, $sc_i \in [0,100]$ for each $1 \le i \le n$, and

$$\sum_{i=1}^{n} sc_i = 100. \tag{14}$$

The SEFRI method consists of five steps as follows:

Step 1 Determine the general fuzzy evaluation of the student's accuracy as weighted average of the individual accuracy values

$$\overline{AC} = \frac{\sum_{i=1}^{n} ac_i \cdot sc_i}{\sum_{i=1}^{n} sc_i}, \tag{15}$$

where the accuracy is weighted by the maximum score values assigned to the individual questions. The calculations are done by the means of the fuzzy sum and multiplication presented in section 2.1.

Step 2 Fuzzyfy the relative time need of the response. We use singleton type fuzzyfication, i.e. a fuzzy set is generated whose membership function is always zero except of the point t_r where its value is 1.

Step 3 Determine the general fuzzy evaluation of the student's correctness in the use of technical terms. It is calculated as weighted average of the individual correctness values

$$\overline{CU} = \frac{\sum_{i=1}^{n} cu_i \cdot sc_i}{\sum_{i=1}^{n} sc_i}, \tag{16}$$

where the correctness is weighted by the maximum score values assigned to the individual questions. The calculations are done by the means of the fuzzy sum and multiplication presented in section 2.1.

Step 4 Determine the general fuzzy evaluation of the student using fuzzy inference. In order to ensure a proper level of granularity we defined five-set partitions (Figure 6) in case of the three evaluation aspects (antecedent dimensions). The antecedent partitions are similar to the ones presented in (Bai and Chen 2008). The fuzzy partition applied in case of the consequent universe of discourse (Figure 5) was determined by the scoring practice of our institution.

A dense rule base covering the whole input space would require $5 \cdot 5 \cdot 5 = 125$ fuzzy rules. In order to reduce the complexity of the system and its storage demand we created a sparse fuzzy rule base containing 64 rules, which are presented in Tables 2, 3, and 4. The abbreviations correspond to the initials of the antecedent and consequent linguistic terms. An example rule is the following:

IF accuracy is high AND relative time need is more or less low AND
correct use of the technical terms is more or less high
THEN general fuzzy evaluation is good.

Owing to the sparse character of the rule base we chose the fuzzy rule interpolation based LESFRI inference technique for the determination of the conclusion (general fuzzy evaluation).

Step 5 Defuzzyfy the general fuzzy evaluation. We apply Center Of Area type defuzzyfication for this task. The resulting crisp value will be the total score of the student from which the grade is determined by the help of Table 1.

4 AN EXAMPLE

In this section, we use an example to illustrate the student evaluation based on fuzzy rule interpolation. In order to facilitate the matrix calculations we use trapezoidal shaped fuzzy sets that can be described by the abscissas of the four break-points supposing that their ordinates are always

$$y = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}^T.$$

They are also able to describe triangle and singleton shaped linguistic terms. Thus the fuzzy partitions used in case of the antecedent dimensions will be described by

$$AS = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.5 & 0.7 \\ 0 & 0.3 & 0.5 & 0.7 & 0.9 \\ 0.1 & 0.3 & 0.5 & 0.7 & 1.0 \\ 0.1 & 0.5 & 0.7 & 0.9 & 1.0 \end{bmatrix}.$$

Furthermore we choose the midpoint of the core as reference point of the sets

$$RP(AS) = \begin{bmatrix} 0.05 & 0.3 & 0.5 & 0.7 & 0.95 \end{bmatrix}.$$

Assume that there are four questions on the answerscripts whose maximal score values are

$$SC = \begin{bmatrix} 30 & 30 & 20 & 20 \end{bmatrix}.$$

Their sum is

$$SCS = 30 + 30 + 20 + 20 = 100 .$$

Assume that the accuracy and the correctness matrices as well as the relative time need of the response are as follows

$$AC = \begin{bmatrix} 0.1 & 0.3 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.5 & 0.7 \\ 0.3 & 0.5 & 0.5 & 0.7 \\ 0.5 & 0.7 & 0.7 & 0.9 \end{bmatrix},$$

$$CU = \begin{bmatrix} 0.3 & 0.3 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.7 & 0.7 \\ 0.5 & 0.5 & 0.7 & 0.7 \\ 0.7 & 0.7 & 0.9 & 0.9 \end{bmatrix},$$

$$t_r = 0.75 .$$

Step 1 We determine the general fuzzy evaluation of the student's accuracy

$$\overline{AC} = AC \cdot SC^T \cdot \frac{1}{SCS} = \begin{bmatrix} 0.1 & 0.3 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.5 & 0.7 \\ 0.3 & 0.5 & 0.5 & 0.7 \\ 0.5 & 0.7 & 0.7 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} 30 \\ 30 \\ 20 \\ 20 \end{bmatrix} \cdot \frac{1}{100} = \begin{bmatrix} 0.28 \\ 0.48 \\ 0.48 \\ 0.68 \end{bmatrix}.$$

The reference point is

$$RP\left(\overline{AC}\right) = 0.48 .$$

Step 2 We fuzzyfy the relative time need of the response. Although we use singleton type fuzzyfication the singleton is still described by a trapezoid.

$$TR = \begin{bmatrix} 0.75 & 0.75 & 0.75 & 0.75 \end{bmatrix}^T$$

The reference point is

$$RP(TR) = 0.75 \, .$$

Step 3 We determine the general fuzzy evaluation of the student's correctness in the use of technical terms

$$\overline{CU} = CU \cdot SC^T \cdot \frac{1}{SCS} = \begin{bmatrix} 0.3 & 0.3 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.7 & 0.7 \\ 0.5 & 0.5 & 0.7 & 0.7 \\ 0.7 & 0.7 & 0.9 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} 30 \\ 30 \\ 20 \\ 20 \end{bmatrix} \cdot \frac{1}{100} = \begin{bmatrix} 0.38 \\ 0.58 \\ 0.58 \\ 0.78 \end{bmatrix} .$$

The reference point is

$$RP\left(\overline{CU}\right) = 0.58 \, .$$

Step 4 We determine the general fuzzy evaluation of the student using LESFRI. First we interpolate a new rule in the position defined by reference points of *AC*, *TR*, and *CU* (also called interpolation points). The antecedent sets of the rule will be calculated in each dimension separately.

In case of the dimension *AC* we calculate first the distance between the interpolation point and the reference points of the existing sets

$$D_{AC} = d\left(RP\left(\overline{AC}\right), RP(AS)\right) = RP\left(\overline{AC}\right) \cdot (1)_{1,5} - RP(AS),$$

$$D_{AC} = 0.48 \cdot (1)_{1,5} - \begin{bmatrix} 0.05 & 0.30 & 0.50 & 0.70 & 0.95 \end{bmatrix} = \begin{bmatrix} 0.43 & 0.18 & -0.02 & -0.22 & -0.47 \end{bmatrix} .$$

Next, we calculate the reciprocal values of the distances' squares in order to get the weights attached to the individual sets

$$W_{AC} = \left[\frac{1}{dAC_{1,j}^2} \right] = \begin{bmatrix} 5.4083 & 30.8642 & 2500 & 20.6612 & 4.5269 \end{bmatrix} .$$

The sum of the weights is

$$SW_{AC} = 2561.4606 \, .$$

After shifting the original sets into the position of the interpolation their new abscissas will be

$$SAS_{AC} = AS + (1)_{4,1} \cdot D_{AC} = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.5 & 0.7 \\ 0 & 0.3 & 0.5 & 0.7 & 0.9 \\ 0.1 & 0.3 & 0.5 & 0.7 & 1.0 \\ 0.1 & 0.5 & 0.7 & 0.9 & 1.0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0.43 \\ 0.18 \\ -0.02 \\ -0.22 \\ -0.47 \end{bmatrix}^T ,$$

$$SAS_{AC} = \begin{bmatrix} 0.43 & 0.28 & 0.28 & 0.28 & 0.23 \\ 0.43 & 0.48 & 0.48 & 0.48 & 0.43 \\ 0.53 & 0.48 & 0.48 & 0.48 & 0.53 \\ 0.73 & 0.68 & 0.68 & 0.68 & 0.53 \end{bmatrix} .$$

From the overlapped set shapes we determine the break-points of the interpolated set

$$AC^i = \frac{SAS_{AC} \cdot W_{AC}^T}{SW_{AC}} = \begin{bmatrix} 0.43 & 0.28 & 0.28 & 0.28 & 0.23 \\ 0.43 & 0.48 & 0.48 & 0.48 & 0.43 \\ 0.53 & 0.48 & 0.48 & 0.48 & 0.53 \\ 0.73 & 0.68 & 0.68 & 0.68 & 0.53 \end{bmatrix} \cdot \begin{bmatrix} 5.4083 \\ 30.8642 \\ 2500 \\ 20.6612 \\ 4.5269 \end{bmatrix} \cdot \frac{1}{2561.4606},$$

$$AC^i = [0.2802 \quad 0.4798 \quad 0.4802 \quad 0.6798]^T.$$

In case of the dimension *TR* we calculate first the distance between the interpolation point and the reference points of the existing sets

$$D_{TR} = d\left(RP\left(\overline{TR}\right), RP(AS)\right) = RP\left(\overline{TR}\right) \cdot (1)_{1,5} - RP(AS),$$

$$D_{TR} = 0.75 \cdot (1)_{1,5} - [0.05 \quad 0.30 \quad 0.50 \quad 0.70 \quad 0.95] = [0.70 \quad 0.45 \quad 0.25 \quad 0.05 \quad -0.20].$$

Next, we calculate the reciprocal values of the distances' squares in order to get the weights attached to the individual sets

$$W_{TR} = \left[\frac{1}{dTR_{1,j}^2}\right] = [2.0408 \quad 4.9383 \quad 16 \quad 400 \quad 25].$$

The sum of the weights is

$$SW_{TR} = 447.9791.$$

After shifting the original sets into the position of the interpolation their new abscissas will be

$$SAS_{TR} = AS + (1)_{4,1} \cdot D_{TR} = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.5 & 0.7 \\ 0 & 0.3 & 0.5 & 0.7 & 0.9 \\ 0.1 & 0.3 & 0.5 & 0.7 & 1.0 \\ 0.1 & 0.5 & 0.7 & 0.9 & 1.0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0.70 \\ 0.45 \\ 0.25 \\ 0.05 \\ -0.20 \end{bmatrix}^T,$$

$$SAS_{TR} = \begin{bmatrix} 0.7 & 0.55 & 0.55 & 0.55 & 0.5 \\ 0.7 & 0.75 & 0.75 & 0.75 & 0.7 \\ 0.8 & 0.75 & 0.75 & 0.75 & 0.8 \\ 1 & 0.95 & 0.95 & 0.95 & 0.8 \end{bmatrix}.$$

From the overlapped set shapes we determine the break-points of the interpolated set

$$TR^i = \frac{SAS_{TR} \cdot W_{TR}^T}{SW_{TR}} = \begin{bmatrix} 0.43 & 0.28 & 0.28 & 0.28 & 0.23 \\ 0.43 & 0.48 & 0.48 & 0.48 & 0.43 \\ 0.53 & 0.48 & 0.48 & 0.48 & 0.53 \\ 0.73 & 0.68 & 0.68 & 0.68 & 0.53 \end{bmatrix} \cdot \begin{bmatrix} 2.0408 \\ 4.9383 \\ 16 \\ 400 \\ 25 \end{bmatrix} \cdot \frac{1}{447.9791},$$

$$TR^i = [0.5479 \quad 0.7470 \quad 0.7530 \quad 0.9419]^T.$$

In case of the dimension *CU* we calculate first the distance between the interpolation point and the reference points of the existing sets

$$D_{CU} = d\left(RP\left(\overline{CU}\right), RP\left(AS\right)\right) = RP\left(\overline{CU}\right) \cdot (1)_{1,5} - RP(AS),$$

$$D_{CU} = 0.58 \cdot (1)_{1,5} - \begin{bmatrix} 0.05 & 0.30 & 0.50 & 0.70 & 0.95 \end{bmatrix} = \begin{bmatrix} 0.53 & 0.28 & 0.08 & -0.12 & -0.37 \end{bmatrix}.$$

Next, we calculate the reciprocal values of the distances' squares in order to get the weights attached to the individual sets

$$W_{CU} = \left[\frac{1}{dCU_{1,j}^2} \right] = \begin{bmatrix} 3.56 & 12.7551 & 156.25 & 69.4444 & 7.3046 \end{bmatrix}.$$

The sum of the weights is

$$SW_{CU} = 249.3141 .$$

After shifting the original sets into the position of the interpolation their new abscissas will be

$$SAS_{CU} = AS + (1)_{4,1} \cdot D_{CU} = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.5 & 0.7 \\ 0 & 0.3 & 0.5 & 0.7 & 0.9 \\ 0.1 & 0.3 & 0.5 & 0.7 & 1.0 \\ 0.1 & 0.5 & 0.7 & 0.9 & 1.0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0.53 \\ 0.28 \\ 0.08 \\ -0.12 \\ -0.37 \end{bmatrix}^T ,$$

$$SAS_{CU} = \begin{bmatrix} 0.53 & 0.38 & 0.38 & 0.38 & 0.33 \\ 0.53 & 0.58 & 0.58 & 0.58 & 0.53 \\ 0.63 & 0.58 & 0.58 & 0.58 & 0.63 \\ 0.83 & 0.78 & 0.78 & 0.78 & 0.63 \end{bmatrix}.$$

From the overlapped set shapes we determine the break-points of the interpolated set

$$CU^i = \frac{SAS_{CU} \cdot W_{CU}^T}{SW_{CU}} = \begin{bmatrix} 0.53 & 0.38 & 0.38 & 0.38 & 0.33 \\ 0.53 & 0.58 & 0.58 & 0.58 & 0.53 \\ 0.63 & 0.58 & 0.58 & 0.58 & 0.63 \\ 0.83 & 0.78 & 0.78 & 0.78 & 0.63 \end{bmatrix} \cdot \begin{bmatrix} 3.5600 \\ 12.7551 \\ 156.2500 \\ 69.4444 \\ 7.3046 \end{bmatrix} \cdot \frac{1}{249.3141} ,$$

$$CU^i = \begin{bmatrix} 0.3807 & 0.5778 & 0.5822 & 0.7763 \end{bmatrix}^T .$$

Assume that the rules are stored in a 64x4 matrix that contains the ordinal numbers of the sets referenced in the rules. It also can be described by

$$R = \begin{bmatrix} A & C \end{bmatrix},$$

where $A = (a_{ij})$ is a 64x3 matrix containing the ordinal numbers of the antecedent sets and $C = (c_i)$ is a column vector with 64 elements containing the ordinal numbers of the consequent sets. Owing to the size of *R* and the derived matrices further on in same cases only the general form of the equation will be presented. Assume that the reference points of these sets are stored in the matrices RP_A and RP_B that are also of size 64x3 and 64x1 respectively. By the help of RP_A each rule can be represented with a point in the antecedent space. Let us call it shortly representative point of the rule. Denote by

$$x^i = \begin{bmatrix} RP\left(AC^i\right) \\ RP\left(TR^i\right) \\ RP\left(CU^i\right) \end{bmatrix} = \begin{bmatrix} 0.48 \\ 0.75 \\ 0.58 \end{bmatrix}$$

the multidimensional interpolation point in the antecedent space.

Next, we determine the square distance between the interpolation point and the representative points of the rules

$$d = \left((1)_{64,1} \cdot \left(x^i\right)^T - RP_A \right) \circ \left((1)_{64,1} \cdot \left(x^i\right)^T - RP_A \right),$$

where $\circ$ denotes the Hadamar product. Each row of the matrix *d* contains the distance between a rule antecedent and the interpolation point measured in an antecedent dimension. The resulting square distance will be the sum of these

$$D = \left(\sum_{j=1}^{3} d_{ij} \right),$$

where *D* is column vector with 64 elements. If the interpolation point is identical with one of the representative points of the rules the position of the interpolated consequent set will be identical with the position of the rule's consequent set. Otherwise we calculate its position as follows. First we determine the weights associated to the rules

$$W = (w_i) = (1)_{64,1} \, (/) \, D \, ,$$

where $(/)$ denotes the element-by-element wise division. The reference point of the consequent linguistic term of the interpolated rule is

$$RP\left(B^i\right) = \frac{\sum_{i=1}^{64} c_i \cdot w_i}{\sum_{i=1}^{64} w_i} = 69.8307 \, .$$

We calculate the shape of the consequent set of the interpolated rule using the same set interpolation technique like in the case of the antecedent sets. The original fuzzy partition used in case of the consequent dimension conform Table 1 will be described by

$$BS = \begin{bmatrix} 0 & 51 & 61 & 76 & 86 \\ 0 & 51 & 61 & 76 & 86 \\ 51 & 61 & 76 & 86 & 100 \\ 51 & 61 & 76 & 86 & 100 \end{bmatrix}.$$

Furthermore we choose the midpoint of the core as reference point of the sets

$$RP(BS) = [25.50 \quad 56 \quad 68.5 \quad 81 \quad 93].$$

We calculate the distance between the interpolation point (the reference point of the consequent linguistic term of the interpolated rule) and the reference points of the existing sets

$$D_B = d\left(RP\left(B^i\right), RP\left(BS\right)\right) = RP\left(B^i\right) \cdot (1)_{1,5} - RP(BS),$$

$$D_B = 69.8307 \cdot (1)_{1,5} - [25.50 \quad 56 \quad 68.5 \quad 81 \quad 93] = [44.3307 \quad 13.8307 \quad 1.3307 \quad -11.1693 \quad -23.1693].$$

Next, we calculate the reciprocal values of the distances' squares in order to get the weights attached to the individual sets

$$W_B = \left[\frac{1}{dB_{1,j}^2}\right] = [0.0005 \quad 0.0052 \quad 0.5647 \quad 0.0080 \quad 0.0019].$$

The sum of the weights is

$$SW_B = 0.5804 .$$

After shifting the original sets into the position of the interpolation their new abscissas will be

$$SBS_B = BS + (1)_{4,1} \cdot D_B = \begin{bmatrix} 0 & 51 & 61 & 76 & 86 \\ 0 & 51 & 61 & 76 & 86 \\ 51 & 61 & 76 & 86 & 100 \\ 51 & 61 & 76 & 86 & 100 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 44.3307 \\ 13.8307 \\ 1.3307 \\ -11.1693 \\ -23.1693 \end{bmatrix}^T ,$$

$$SBA_B = \begin{bmatrix} 44.3307 & 64.8307 & 62.3307 & 64.8307 & 62.8307 \\ 44.3307 & 64.8307 & 62.3307 & 64.8307 & 62.8307 \\ 95.3307 & 74.8307 & 77.3307 & 74.8307 & 76.8307 \\ 95.3307 & 74.8307 & 77.3307 & 74.8307 & 76.8307 \end{bmatrix}.$$

From the overlapped set shapes we determine the break-points of the interpolated set

$$B^i = \frac{SBA_B \cdot W_B^T}{SW_B} = \begin{bmatrix} 44.3307 & 64.8307 & 62.3307 & 64.8307 & 62.8307 \\ 44.3307 & 64.8307 & 62.3307 & 64.8307 & 62.8307 \\ 95.3307 & 74.8307 & 77.3307 & 74.8307 & 76.8307 \\ 95.3307 & 74.8307 & 77.3307 & 74.8307 & 76.8307 \end{bmatrix} \cdot \begin{bmatrix} 0.0005 \\ 0.0052 \\ 0.5647 \\ 0.0080 \\ 0.0019 \end{bmatrix} \cdot \frac{1}{0.5804} ,$$

$$B^i = [62.3736 \quad 62.3736 \quad 77.2878 \quad 77.2878]^T .$$

The last step of LESFRI determines the conclusion by modifying the rule consequent depending on the differences between the rule antecedent sets and the observation (fuzzy evaluation of the student). Denote by

$$A^* = [\overline{AC} \quad TR \quad \overline{CU}] = \begin{bmatrix} 0.28 & 0.75 & 0.38 \\ 0.48 & 0.75 & 0.58 \\ 0.48 & 0.75 & 0.58 \\ 0.68 & 0.75 & 0.78 \end{bmatrix}$$

the matrix representing the fuzzy evaluation of the student. Denote by

$$A^i = [AC^i \quad TR^i \quad CU^i] = \begin{bmatrix} 0.2802 & 0.5479 & 0.3807 \\ 0.4708 & 0.7470 & 0.5778 \\ 0.4802 & 0.7530 & 0.5822 \\ 0.6798 & 0.9419 & 0.7763 \end{bmatrix}$$

the antecedent of the interpolated rule. The deviation between them is

$$D_A = A^i - A^* = \begin{bmatrix} 0.2802 & 0.5479 & 0.3807 \\ 0.4798 & 0.7470 & 0.5778 \\ 0.4802 & 0.7530 & 0.5822 \\ 0.6798 & 0.9419 & 0.7763 \end{bmatrix} - \begin{bmatrix} 0.28 & 0.75 & 0.38 \\ 0.48 & 0.75 & 0.58 \\ 0.48 & 0.75 & 0.58 \\ 0.68 & 0.75 & 0.78 \end{bmatrix} = \begin{bmatrix} 0.0002 & -0.2021 & 0.0007 \\ -0.0002 & -0.0030 & -0.0022 \\ 0.0002 & 0.0030 & 0.0022 \\ -0.0002 & 0.1919 & -0.0037 \end{bmatrix} = \left(dA_{ij}\right).$$

The mean deviation for each α-level is

$$MD = \left(\frac{\sum_{j=1}^{3} dA_{ij}}{3} \right) = \begin{bmatrix} -0.0671 \\ -0.0018 \\ 0.0018 \\ 0.0627 \end{bmatrix}.$$

The raw shape of the conclusion set is

$$B^* = B^i - MD = \begin{bmatrix} 62.3736 \\ 62.3736 \\ 77.2878 \\ 77.2878 \end{bmatrix} - \begin{bmatrix} -0.0671 \\ -0.0018 \\ 0.0018 \\ 0.0627 \end{bmatrix} = \begin{bmatrix} 62.4406 \\ 62.3754 \\ 77.2860 \\ 77.2251 \end{bmatrix} = \left(b_i^*\right)$$

The reference point of the conclusion set is

$$RP\left(B^*\right) = \frac{b_2^* + b_3^*}{2} = \frac{62.3754 + 77.2860}{2} = 69.8307 \text{ ,}$$

which is identical with the reference point of the conclusion set of the interpolated rule and therefore the core of B^{st} is adequate. However the support is abnormal, and it needs a correction

$$b_1^* = min\left(b_1^*, b_2^*\right) = 62.3754 \text{ ,}$$

$$b_4^* = max\left(b_4^*, b_5^*\right) = 77.2860 \text{ .}$$

Thus the final shape of the conclusion is defined by

$$B^* = \left[62.3754 \quad 62.3754 \quad 77.2860 \quad 77.2860\right]^T .$$

The total score of the student is obtained by COA defuzzyfication, which will result in this case

$$TS = \frac{62.3754 + 62.3754 + 77.2860 + 77.2860}{4} = 69.8307 \text{ .}$$

Conform Table 1 the grade of the student is “Average”.

CONCLUSIONS

Student evaluation in case of tests requiring narrative responses contains a lot of steps that are from nature fuzzy owing to the uncertainties in the judgment of the evaluators, which cause that they cannot express the evaluation with only one crisp number. In this paper, a new method called SEFRI was introduced aiming the support of the evaluation by allowing the use of three main aspects, i.e. the accuracy of the response, the correctness of the technical terms’ use, and the total time necessary for answering the questions, as well as the expression of the uncertainty of the evaluator’s opinion in form of fuzzy numbers.

The final evaluation of the students is a result of a fuzzy average calculation and a fuzzy inference that applies a compact knowledge representation using a sparse rule base and a fuzzy rule interpolation based reasoning method. Similar to other fuzzy set theory based approaches SEFRI can be used in practice only if the software background is ensured. The Matlab implementation of the method is supported by the FRI ToolBox (Johanyák et al. 2006) that can be downloaded freely from http://fri.gamf.hu. The rule base of the method was generated "by hand" based on expert knowledge gained from evaluators with several years experience. Further research plans include the automatic generation of the rule base from sample data using for example the methods presented in (Botzheim et al 2001), (Gál and Kóczy 2008) or (Škrjanc and Blažič 2005) as well as the application of a hierarchical fuzzy inference system (Hladek et al. 2008).

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REFERENCES

Bai, S.M., and Chen, S. M., 2008, *Evaluating students' learning achievement using fuzzy membership functions and fuzzy rules*, Expert Systems with Applications, 34, 399-410.

Biswas, R., 1995, *An application of fuzzy sets in students' evaluation*, Fuzzy Sets and Systems, 74(2), 187-194.

Botzheim, J., Hámori, B., and Kóczy, L.T., 2001, *Extracting trapezoidal membership functions of a fuzzy rule system by bacterial algorithm*, in Proceedings of the International Conference 7th Fuzzy Days, Dortmund, Germany, Lecture Notes in Computer Science, Springer-Verlag, 2206, 218-227.

Chen, S. M., and Ko, Y. K., 2008, *Fuzzy Interpolative Reasoning for Sparse Fuzzy Rule-Based Systems Based on α-cuts and Transformations techniques*, IEEE Transactions on Fuzzy Systems, Vol. 16, No. 6, 1626-1648.

Chen, S. M., and Lee, C.H., 1999, *New methods for students' evaluating using fuzzy sets*, Fuzzy Sets and Systems, 104(2), 209-218.

Fodor, J., and Bede, B., 2006, *Arithmetics with fuzzy numbers: a comparative overview*, in Proceedings of the 4th Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence (SAMI 2006), Herlany, Slovakia, 54-68.

Gál, L., and Kóczy, L.T., 2008, *Advanced Bacterial Memetic Algorithms*, Acta Technica Jaurinensis, Series Intelligentia Computatorica, 1(3), 225-243.

Hládek, D., Vaščák, J., and Sinčák, P., 2008, *Hierarchical fuzzy inference system for robotic pursuit evasion task*, in Proc. of the 6th International Symposium on Applied Machine Intelligence and Informatics (SAMI 2008), January 21-22, Herľany, Slovakia, 273-277.

Johanyák, Z.C., and Kovács, S., 2006, *Fuzzy Rule Interpolation by the Least Squares Method*, in Proceedings of the 7th International Symposium of Hungarian Researchers on Computational Intelligence (HUCI 2006), Budapest, Hungary, 495-506.

Johanyák, Z.C., Tikk, D., Kovács, and S., Wong, K.K., 2006, *Fuzzy Rule Interpolation Matlab Toolbox - FRI Toolbox*, in Proceedings of the IEEE World Congress on Computational Intelligence (WCCI'06), 15th International Conference on Fuzzy Systems (FUZZ-IEEE'06), Vancouver, BC, Canada, 1427-1433.

Kóczy, L.T., and Hirota, K., 1993, *Approximate reasoning by linear rule interpolation and general approximation*, International Journal of Approximate Reasoning, 9, 197-225.

Kovács, S., 2006, *Extending the Fuzzy Rule Interpolation "FIVE" by Fuzzy Observation*, Computational Intelligence, Theory and Applications, Springer-Verlag, Berlin, Heidelberg, 2006, pp. 485-497.

Nolan, J. R., 1998, *An expert fuzzy classification system for supporting the grading of student writing samples*, Expert Systems With Applications, 15, 59-68.

Precup, R.E., Preitl, S., Tar, J.K., Tomescu, M.L., Takács, M., Korondi, P., and Baranyi, P., 2008, *Fuzzy control system performance enhancement by Iterative Learning Control*, IEEE Transactions on Industrial Electronics, 5(9), 3461-3475.

Shepard, D., 1968, *A two dimensional interpolation function for irregularly spaced data*, in Proceedings of the 23rd ACM International Conference, New York, USA, 517-524.

Škrjanc, I., Blažič, S., and Agamennoni, O.E., 2005, *Interval fuzzy model identification using* l_∞*-norm*, IEEE Transactions on Fuzzy Systems, 13(5), 561-568.

Tikk, D., and Baranyi, P., 2000, *Comprehensive analysis of a new fuzzy rule interpolation method*, in IEEE Transactions on Fuzzy Systems, 8, 281-296.

Tikk, D., Kóczy, L.T., and Gedeon T.D., 2003, *A survey on the universal approximation and its limits in soft computing techniques*, International Journal of Approximate Reasoning, 33, 185-202.

Wang, H.Y., and Chen, S.M., 2006, *New methods for Evaluating the Answerscripts of Students Using Fuzzy Sets*, Advances in Applied Artificial Intelligence, Lecture Notes in Computer Science, Springer-Verlag, 4031, 442-451.

TABLES AND FIGURES

Table 1: Relation between scores and grades.

Score intervals	Grades
0 - 50	Unsatisfactory
51 - 60	Satisfactory
61 - 75	Average
76 - 85	Good
86 - 100	Excellent

Table 2: Fuzzy rules for AC=Low and AC=More or Less Low.

	AC=L					AC=MLL				
TR / CU	L	MLL	M	MLH	H	L	MLL	M	MLH	H
L	U		U		S		S		S	
MLL						S		S		S
M			S							
MLH						S		S		A
H	S		S		S		S		A	

Table 3: Fuzzy rules for AC=Medium and AC=More or Less High.

	AC=M					AC=MLH				
TR / CU	L	MLL	M	MLH	H	L	MLL	M	MLH	H
L	S		A		A	A		A	G	
MLL		A		A	A		A	G		G
M			A	A		A	G			
MLH		A	A		A	G			G	G
H	A	A		A	G		G		G	G

Table 4: Fuzzy rules for AC=High.

TR / CU	L	MLL	M	MLH	H
L	G	G	G	G	G
MLL		G	G		E
M	G			E	
MLH	G	G	E	E	
H	G	E	E		E

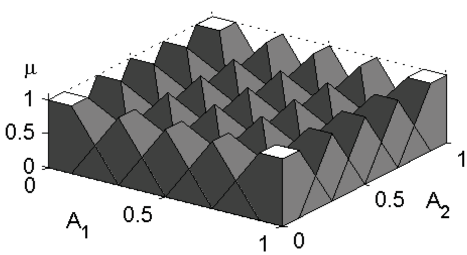

Figure 1: Dense fuzzy rule base in case of a two dimensional antecedent universe of discourse.

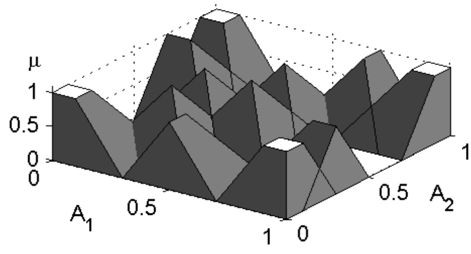

Figure 2: Sparse fuzzy rule base in case of a two dimensional antecedent universe of discourse.

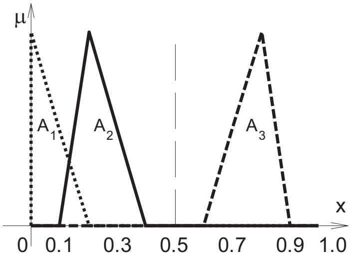

Figure 3: Original partition

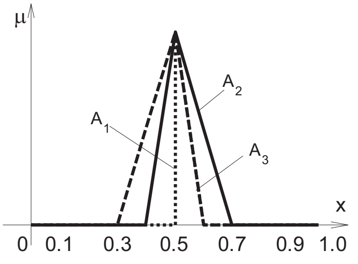

Figure 4: Partition with shifted sets

Figure 5: Traditional grades represented as crisp fuzzy sets.

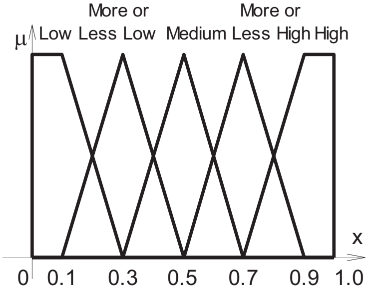

Figure 6: Fuzzy partition used in case of the antecedent dimensions.