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Vague Environment Based Set Interpolation

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Fuzzy Set Interpolation aims the generation of a new linguistic term in a given position of a fuzzy partition taking into consideration the shape and the position of the neighbouring sets. It is applied in the first step of the Generalized Methodology (GM) [1] of Fuzzy Rule Interpolation (FRI) for the calculation of the antecedent and consequent parts of the new rule.

This paper presents a new FSI method called Vague Environment based Set Interpolation (VESI), which synthesizes the benefits of the concepts Vague Environment (VR) [8][11] and GM. It ensures the desired speed and the ability to handle non-singleton observations and conclusions for the FRI technique based on it.

1. Introduction

Systems based on fuzzy reasoning present a good solution for cases where a mapping between the observation and the resulting action either cannot be described in form of one or more exact mathematical equations or the determination of the equations would be too complicated.

The selection of the applied inference technique is determined primarily by the availability of the whole bulk of the rules necessary for the coverage i.e. dense character of the rule base. Having a sparse rule base (see fig. 1) in lack of the coverage in case of some observations (e.g. A^*, B^*) there are no rules whose antecedent part would overlap the observation at least partially. Therefore approximate techniques are needed for the calculation of the conclusion. Usually these techniques are based on Fuzzy Rule Interpolation (FRI).

The FRI methods can be classified into two groups depending on whether they are applying the one-step or the two-step approach. The key feature of the first group is that the conclusion is determined directly from the observation and the neighbouring rules taken into consideration. Contrary to this in the case of the second group a new intermediate rule is generated in the position of the observation and the conclusion results from the application (firing) of the auxiliary rule.

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Relevant members of the first group are the α-cut based interpolation (KH) proposed by Kóczy and Hirota in [9], which was the first developed technique, the modified α-cut based interpolation (MACI) [15] introduced by Tikk and Baranyi, the fuzzy interpolation based on vague environment (FIVE) [11] developed by Kovács and Kóczy, its extended version suggested by Kovács in [12], the improved fuzzy interpolation technique for multi-dimensional input spaces (IMUL) [17] proposed by Wong, Gedeon and Tikk, the interpolative reasoning based on graduality (IRG) [2] introduced by Bouchon-Meunier, Marsala and Rifqi, the interpolation by the conservation of fuzziness (GK) [3] developed by Gedeon and Kóczy, the method based on the conservation of the relative fuzziness (CRF) [10] proposed by Hirota, Kóczy and Gedeon, and the VKK method [16] introduced by Vass, Kalmár and Kóczy.

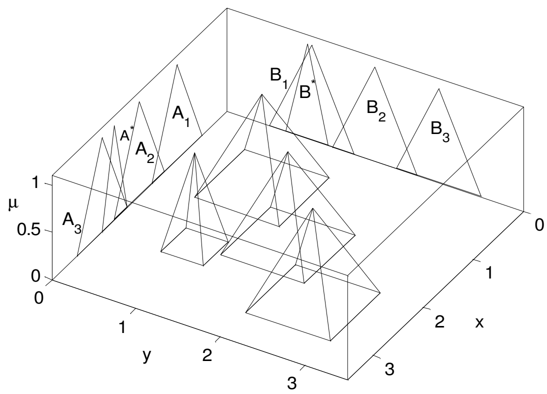

Fig. 1. Sparse rule base

The basic concepts of the second approach were formalized first in the Generalized Methodology of Fuzzy Rule Interpolation (GM) introduced by Baranyi, Kóczy and Gedeon in [1]. As other typical members of this group can be mentioned the ST method [18] introduced by Yan, Mizumoto and Qiao, the Interpolation with Generalized Representative Values (IGRV) [4] developed by Huang and Shen, the technique proposed by Jenei in [5], and the method FRIPOC introduced by Johanyák and Kovács in [6].

The solvability and approximate solvability of fuzzy relation equations and the approximation quality of approximate solutions was studied by Perfilieva and Gottwald in [14].

Several FRI techniques suffer from a common drawback, namely the high computational demand. This feature makes difficult their use in embedded systems or real time applications [12]. This recognition has led to the development of the inference method FIVE [11]. It alleviated the problem by the application of the concept Vague Environment (VE), which enables a huge amount of the calculations to be done in advance before the reasoning itself. However, this method has some weak points as well. It enables only singleton type observation shapes and the conclusion is always a singleton. A solution for the first problem was introduced in [12], which is based on the concept of merging VEs. Although its benefits the applicability of the extension is not universal.

In this paper a new approach is introduced based on the concept Vague Environment and on the Generalized Methodology (GM). The method Vague Environment based Set Interpolation (VESI) is a fuzzy set interpolation method applicable in the first step of any FRI techniques based on the concepts defined in GM [1]. Its main advantage is that it unifies the benefits of FIVE and GM being quick and enabling a wider range of observation shape types. Besides the shape of the conclusion sets generated by the FRI method based on VESI is not restricted to the singleton case.

The rest of this paper is organized as follows. Section 2 presents the main ideas of GM. Section 3 recalls the basic concepts regarding to the VE and its applicability for fuzzy partition description. Section 4 introduces the method VESI. The steps of the algorithm are presented by a numerical example.

2. Generalized Methodology

The GM was introduced by Baranyi et al in [1]. It divides the task of rule interpolation into two steps. First it interpolates a new rule corresponding to the position of the observation. GM uses a reference point for the characterization of the location of the linguistic terms. Some typical choices for its selection are presented on figure 2.

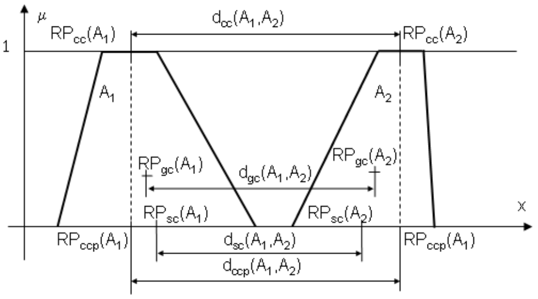

Fig. 2. Choices for the selection of the reference point and the related distances

The distances between the sets are calculated as horizontal distances between the reference points. The new rule is determined in three stages.

1. The antecedent sets are calculated in each dimension by a set interpolation technique.
2. The position of the consequent sets is calculated in each consequent dimension using a crisp interpolation technique.
3. The shape of the consequent sets is determined by using the same set interpolation technique as in stage 1.

The conclusion, the output of the reasoning process is produced in the second step of the GM. Usually the antecedent part of the new rule does not fit perfectly the observation. Therefore a special Single Rule Reasoning (SRR) technique is needed for the calculations.

3. Vague Environment

The concept Vague Environment (VE) was introduced originally by Klawonn in [8] and it was extended and adapted for the application field FRI by Kovács and Kóczy [11]. This approach is based on the similarity or indistinguishability of the elements. Two values of the VE are ε-indistinguishable if their distance is smaller or equal to ε

$$\varepsilon \geq \delta_s(x_1, x_2) = \left| \int_{x_2}^{x_1} s(x)dx \right|, \tag{1}$$

where $s(x)$ is the scaling function, δ_s is the scaled distance, x_1 and x_2 are the two values of the VE. This distance is a weighted proximity measure. The weighting factor is the so called scaling factor.

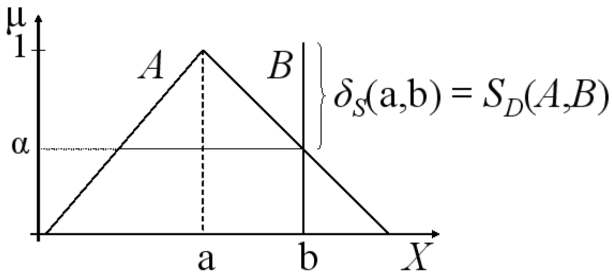

Fig. 3. Disconsistency measure of a triangle shaped fuzzy set an a singleton [12]

The connection between the fuzzy universe and the VE is given by the equality between the disconsistency measure (fig. 3) of two linguistic terms A and B, which is one of the well known similarity measures of fuzzy sets, and the scaled distance $\delta_s(a,b)$ between the prototypical points a and b representing the linguistic terms in the VE of their partition. Due to the nature of the VE only the indistinguishability of a fuzzy set and a singleton can be measured in it. This is why the application area of the original version of FIVE was restricted to singleton shaped observations.

The key point in the application of the concept VE is to find a universal scaling factor that describes the shapes of the linguistic terms of the partition with only one function. Exact scaling functions of the VE can be determined only for some regular cases. For example fig. 4 shows a fuzzy partition containing only one isosceles triangle shaped linguistic term.

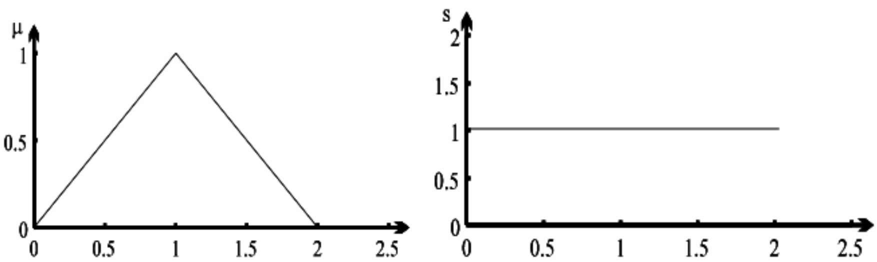

Fig. 4. Simple partition with one isosceles triangle shaped linguistic term and its scaling function [13]

The scaling function corresponding to this set is a horizontal line using the scaling function (2) proposed by Klawonn [8]

$$s(x) = |\mu'(x)| = \left|\frac{d\mu}{dx}\right|. \tag{2}$$

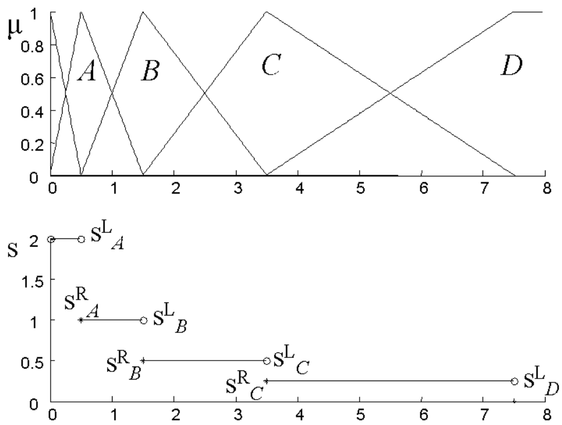

Fig. 5. Ruspini partition and its scaling function [12]

Another example could be a Ruspini partition (fig. 5), where the scaling function consists of a set of horizontal lines corresponding to the individual slopes. However, in the general case approximate scaling functions must be applied for the description of those regions of fuzzy partitions where the exact scaling function cannot be determined due to the intersection or facing (neighbouring) position of rather different set shapes. In such circumstances the non-linear interpolation proposed by Kovács [11] provides the best scaling factor (see fig. 6).

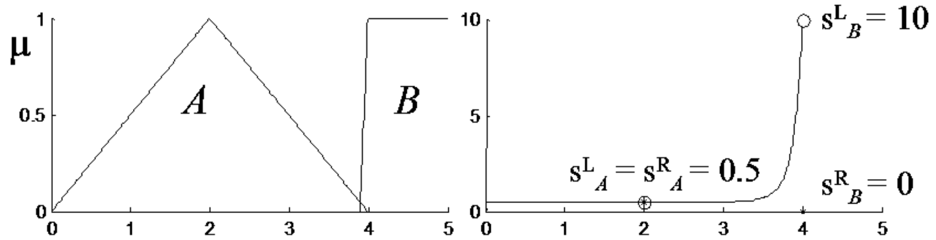

Fig. 6. Partition with different set shapes and its VE using non-linear interpolated scaling function [12]

4. Set interpolation

The method VESI (Vague Environment based Set Interpolation) aims the determination of a new linguistic term in a specific point, called interpolation point, of a fuzzy partition. It can be applied for the determination of the antecedent and consequent sets of the new rule in the first step of an FRI technique, which follows the concepts of the GM [1]. The method is the same regardless of it is applied in case of a rule premise or a rule conclusion. It can be applied in both the cases of sparse and dense partitions.

Similar to the other set or rule interpolation/extrapolation/approximation techniques VESI assumes regularity between the linguistic terms of a fuzzy partition. The first stage of the technique is the generation of the VE of the partition, which has to be done only once, before starting the fuzzy system based on it. Throughout the course of the repetitive reasoning steps the original VE is used in the calculations.

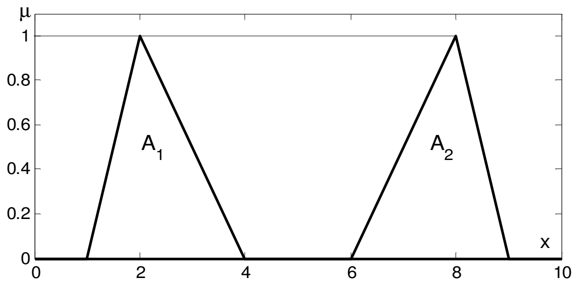

Fig. 7. Sparse partition with two triangle shaped linguistic terms

The second stage of VESI starts with a given interpolation point in the current dimension (partition), which is actually a reference point of a fuzzy set. The basic idea is that the new set conserves the properties of the VE. Thus one can generate the shape of the new linguistic term from the scaling function easily by handling the interpolation point as a prototypical point.

Further on the algorithm is presented by a numerical example. For simplicity and lucidity the sample partition (fig. 7) is sparse and it contains two CNF sets that are triangle shaped. The linguistic terms are defined as follows:

$$A_1 = \{1/0, 2/1, 4/0\}, \tag{3}$$

$$A_2 = \{6/0, 8/1, 9/0\}. \tag{4}$$

The reference (prototypical) points of the sets are the peak points of the triangles. At this time for the sake of simplicity the right flank of the set A_1 and the left flank of the set A_2 have the same slope in absolute value. The range of the partition is $R_A = [0,10]$. The scaling function builds up from the scaling factors (5)..(8) corresponding to the four flanks of the sets

$$s_{A_1}^L = \left| \frac{d\mu_{A_1^L}}{dx} \right| = \frac{1}{1} = 1, \tag{5}$$

$$s_{A_1}^R = \left| \frac{d\mu_{A_1^R}}{dx} \right| = \frac{1}{2} = 0.5, \tag{6}$$

$$s_{A_2}^L = \left| \frac{d\mu_{A_2^L}}{dx} \right| = \frac{1}{2} = 0.5, \tag{7}$$

$$s_{A_2}^R = \left| \frac{d\mu_{A_2^R}}{dx} \right| = \frac{1}{1} = 1, \tag{8}$$

where s_X^Z is the scaling factor corresponding to the *Z* (left or right) flank of the fuzzy set *X* (A_1 or A_2).

Conform to the extended concept of the VE [11][12] each interval of the scaling function delimited by prototypical points – including here also the sparse portions of the partition – is defined only by the neighbouring flanks of the sets. Thus the interval [4,6) is characterized by the same scaling factor $s=0.5$ as its surrounding intervals [2,4) and [6,8). In a similar way in case of the leading and trailing empty (sparse) portions of the partition the scaling factors of the closest set flanks are used for the generation of the VE. Thus the scaling factor corresponding to the intervals [0,1) and [9,10) is $s=1$. This feature ensures the capability of interpolation and extrapolation for the VE. The whole scaling function is described by

$$s(x) = 1(x) - \frac{1}{2} \cdot 1(x-2) + \frac{1}{2} \cdot 1(x-8), \tag{9}$$

where $1(x)$ is the Heaviside (unit step) function. Figure 8 presents the description of the partition in the VE.

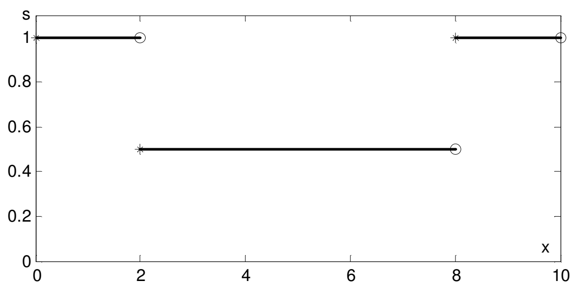

Fig. 8. Vague Environment of the partition

The interpolation point is given by the coordinate pair (5,1). The reference point of the new linguistic term will be identical with this point $A_{rp}^i = (5,1)$. The shape of the new set is calculated as follows. The left flank is determined by the scaling function $s(x)\big|_{x \le A_{rpx}^i}$. Thus $\mu_{A^i}(4) = 0$. The right flank is determined in a similar way by the scaling function $s(x)\Big|_{x \geq A^{i}_{rpx}}$. Thus $\mu_{A^i}(6) = 0$. The resulting partition is presented on figure 9.

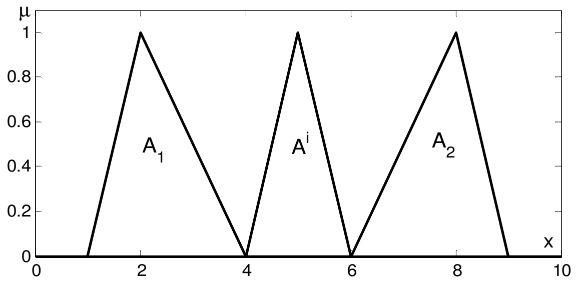

Fig. 9. The resulting partition after the interpolation

VESI fulfils most of the requirements defined in [7] as General conditions on FRI techniques. Some relevant features are listed below.

- The new set never can be an abnormal one due to the nature of the VE and to the algorithm that calculates the two flanks separately and joins them in the interpolation point.
- VESI preserves the "in between" feature because the reference point of the new set is identical with the point of the interpolation.
- The condition "compatibility with the rule base" is only fulfiled when the neighbouring flanks have the same slope in absolute value.
- The fuzziness of the result depends on the shape of the neighbouring flanks of the surrounding sets only in the simplest case (see fig. 7-9). Having an interpolation point in the closest neighbourhood of the reference point of a linguistic term the other flank of the sets also exercises influence on the calculations. Therefore the shape of the obtained set can contain some break-points.
- As a consequence of the above described feature VESI not always conserves the piece-wise linearity of the surrounding sets.
- The FRI method based on it easily can handle multidimensional antecedent universes as well.

4. Conclusions

Fuzzy Rule Interpolation methods are used for reasoning in most of the cases in fuzzy systems built on sparse rule bases. This paper introduces a new approach by applying the concept Vague Environment in an FRI method that follows the Generalized Methodology of FRI.

The presented technique is able to determine the antecedent and consequent sets of the auxiliary rule in the first step of GM. Its main advantages are (1) its low computational complexity resulting from the application of the concept VE, (2) the FRI technique based on VESI will also be able to handle non-singleton observations and (3) the shape of the conclusion sets generated by this FRI technique will not be restricted to the singleton type.

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Halmaz-interpoláció bizonytalan környezetben

Johanyák Zsolt Csaba

Összefoglalás

A fuzzy halmaz-interpoláció célja egy új nyelvi érték előállítása a partíció egy adott pontjában a figyelembe vett környező halmazok elhelyezkedéséből és alakjából kiindulva. Alkalmazására a ritka szabálybázisok esetén használt általánosított szabály-interpolációs módszertan [1] alapú következtetés első lépésében kerül sor.

A bemutatott eljárás a bizonytalan környezet [8][11] fogalmára épülő skálafüggvény segítségével jellemzi a partíciót, majd az interpolációs pont helyzetéből kiindulva a skálafüggvény alapján generál egy új halmazt. Az eljárás előnye gyorsaságában és egyszerűségében rejlik.

Fuzzy Menge-Interpolation in der Vagen Umgebung

Johanyák, Zsolt Csaba

Zusammenfassung

Fuzzy Menge-Interpolation bezweckt die Generierung einem neuen linguistischen Term in einem Punkt der Partition ausgehend von der Position und der Gestalt den umliegenden Mengen. Sie wird im Fall der spärlichen Fuzzy Regelbasis verwendet, um die Prämisse und die Konklusion der neue Regel im ersten Schritt der Generalisierten Methodik für Fuzzy Regel-Interpolation [1] zu produzieren.

Die Vorgestellte Methode beschreibt die Partition mittelst der Skalierfunktion, welche ist auf dem Konzept Vage Umgebung [8][11] basiert. Sie errechnet eine neue Menge ausgehend von der Position der Interpolation und von der Skalierfunktion. Der Vorteil dieser Methode liegt in ihrer Schnelligkeit und Einfachheit.