Fuzzy Rule Interpolation based on Subsethood Values

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***Abstract*—Fuzzy rule interpolation methods make possible the development of fuzzy rule based systems applying a low complexity and compact rule base that contains only the most relevant rules. They are able to infer even in those regions of the antecedent space where there are no applicable rules. In this paper, we present a novel method called Fuzzy Rule Interpolation based on Subsethood Values (FRISUV) that calculates the conclusion directly from the observation and the known rules with a low complexity algorithm.**

***Keywords*—fuzzy rule interpolation, subsethood value**

I. Introduction

Soft computing methods offer solutions for problems that hardly can be solved with the classical approaches owing to the lack of adequate mathematical model or to the high complexity of the existing models. Fuzzy rule based systems became popular due to their self explanatory capability. An IF-THEN structure is easy to understand for everyone and the process called fuzzy inference is close to the human thinking.

However, a disadvantage of the classical compositional fuzzy reasoning techniques (e.g. [13][14][20]) became their demand on a full coverage of the input space by rules. This requirement can be only fulfilled with a high number of rules, which number increases exponentially with the number of linguistic values per antecedent dimension and the number of dimensions, which could make difficult their application in real time or embedded systems (e.g. Lego robots [23]).

The recognition of this problem led in the early 1990s (e.g. [9]) to the emergence of inference methods capable of dealing with sparse rule bases, i.e. rule bases containing only the relevant rules. These approximate reasoning techniques usually apply fuzzy rule interpolation (FRI) methods in order to calculate interpretable results even in regions of the antecedent space where the rule base does not contain any applicable rules.

The FRI methods form two main groups: the one-step and the two-step methods. The FRI techniques belonging to the first group calculate the conclusion directly from the observation and the known rules of the rule base. The most relevant representatives of this group are the linear interpolation that is also called KH method after its developers (Kóczy and Hirota) [9] and which also proved to be an universal function approximator [22], the vague environment based reasoning FIVE developed by Kovács [12], the modified α-cut based interpolation (MACI) published by Tikk and Baranyi that avoids the abnormal conclusion by applying a coordinate transformation [21], the CRF (Kóczy, Hirota, and Gedeon) [10] whose key idea is the conservation of the relative fuzziness, the IMUL method suggested by Wong, Tikk, Gedeon and Kóczy [24] that combines the advantages of the MACI and CRM approaches, the interpolation method developed by Kovács [11] that extended the fuzzy interpolation to the general metric spaces, the method of Dubois and Prade [5] as well as the method proposed by Bouchon-Meunier, Marsala, and Rifqi [3].

The members of the second group determine the conclusion in two steps by first interpolating a new rule in the position corresponding to the observation and next, they infer the result using only the observation and the new rule applying a single rule reasoning technique. These methods follow the concept of the Generalized Methodology of fuzzy rule interpolation (GM) developed by Baranyi, Kóczy and Gedeon [1]. Relevant members of this group are the technique family suggested in [1], the LESFRI (Johanyák and Kovács) [7] that applies the method of least squares for the calculation of the membership functions, the IGRV developed by Huang and Shen [6], the transformation based technique published by Chen and Ko [4] as well as the polar cut based FRIPOC suggested by Johanyák and Kovács in [8].

In spite of the relative high number of existing FRI methods one can hardly find a technique with low computational complexity and applicability in case of all the valid (convex and normal) fuzzy (CNF) sets. This shortcoming makes difficult the application of FRI methods in real-time or embedded systems.

In order to alleviate the above mentioned problem this paper introduces a new FRI method that extends the concept of the fuzzy subsethood values [17] and applies it in conjunction with a relative distance aiming the measurement of the similarity between rule antecedents and observations. Later the conclusion is calculated based on this similarity. The rest of this paper is organized as follows. Section II reviews the concept of sparse rule bases. Section III introduces the new similarity measure and the FRI method based on it. A numerical example is presented in section IV and the conclusions are drawn in section V.

II. Dense and sparse rule bases

A fuzzy rule base is dense or covering when for all the possible observations it exists at least one fuzzy rule whose antecedent part overlaps the observation at least partially.

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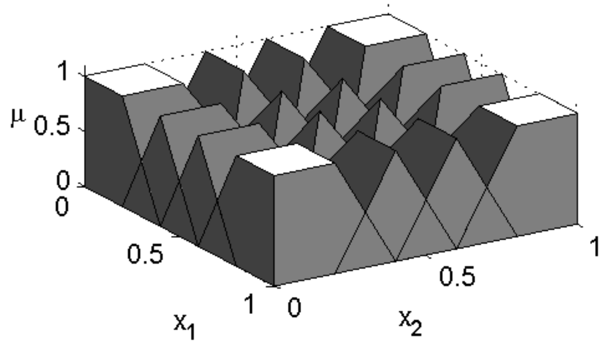

Figure 1. Antecedent space of a dense rule base

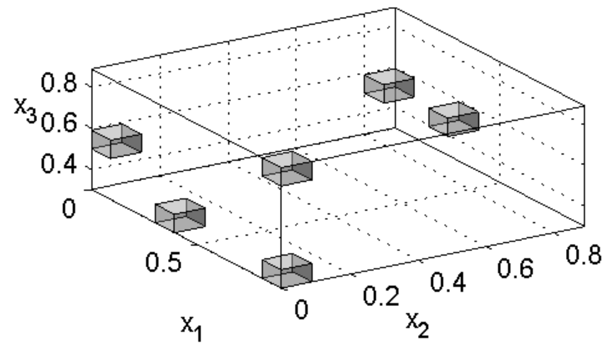

Figure 3. Antecedent space of a sparse fuzzy rule base

For example Fig. 1 presents the antecedent space of a dense rule base with two input linguistic variables (x_1 and x_2). Both partitions have a Ruspini character. In the general case one can define a coverage measure of the antecedent space by the formula

$$c = \arg\max_{\varepsilon}\left(\min_{i=1}^{N}\left\{\max_{j=1}^{n_i}\left\{ t\left(A_{ij}, A_i^*\right)\right\}\right\} \geq \varepsilon, \quad \forall A_i^* \subset X_i\right), \varepsilon \in [0,1], \tag{1}$$

where X_i is the ith dimension of the antecedent space, A_i^* is the fuzzy set describing the observation in the ith antecedent dimension, A_{ij} is the jth linguistic term of the ith antecedent dimension, t is an arbitrary t-norm, n_i is the number of the linguistic terms of the ith antecedent dimension, N is the number of the antecedent dimensions, and *argmax(.)* calculates that ε value for which the expression in the parentheses takes its maximum. If $c > \varepsilon_0$ the rule base is called ε_0 covering (dense) otherwise it is considered as sparse. The Ruspini partition based rule base presented in Fig. 1 ensures an $\varepsilon=0.5$ coverage of the input space.

One ensures easily the required dense character of the rule base in case of one or two dimensional antecedent spaces using partitions containing a reduced number of fuzzy sets. However, increasing the number of input linguistic variables or/and the linguistic values in the partitions the demanded coverage of the input space is realizable only at the expense of a huge number of rules (N_R in (2)).

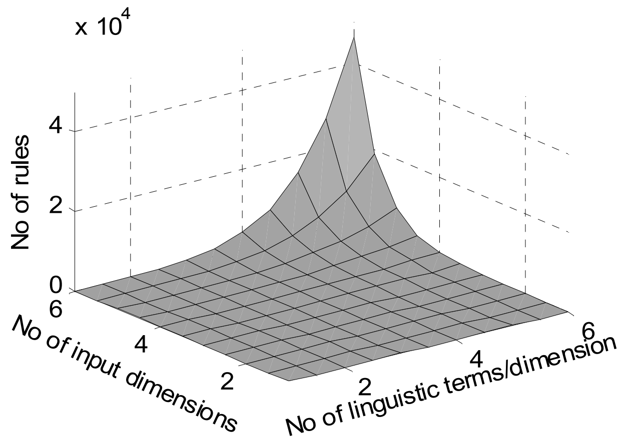

Figure 2. Number of rules in a dense rule base depending on the number of input dimensions and the number of linguistic terms in a dimension

$$N_R = \prod_{i=1}^{N} n_i \tag{2}$$

Fig. 2 illustrates that even by relatively low values of both parameters the number of rules increases exponentially. The rule number explosion leads to increased system complexity and results in grown storage demand and extended time consumption of the output calculation. The problem can be solved by applying sparse rule bases instead of using dense ones. A sparse rule base does not ensure a full coverage of the input space (see Fig. 3), i.e. $\varepsilon=0$.

III. Fuzzy Rule Interpolation based on Subsethood Values

A. Reference Point

The reference point of a fuzzy set ($RP(.)$) is that point of the universe of discourse, which is most characteristic to the set. Generally it is used for the definition of the set's position and for the determination of the distance between the fuzzy sets.

Most often applied choices for its selection are the centre of the core (RP_{CC}) [1][3][7][8], the centre of the support (RP_{SC}) [3], the centre of gravity (RP_{GC}) [6] and the unweighted or weighted average of the abscissas of the characteristic (break) points of the shape (RP_{UAV}, RP_{WAV}) [6]. Fig. 4 presents the listed reference point types in case of a trapezoid shaped fuzzy

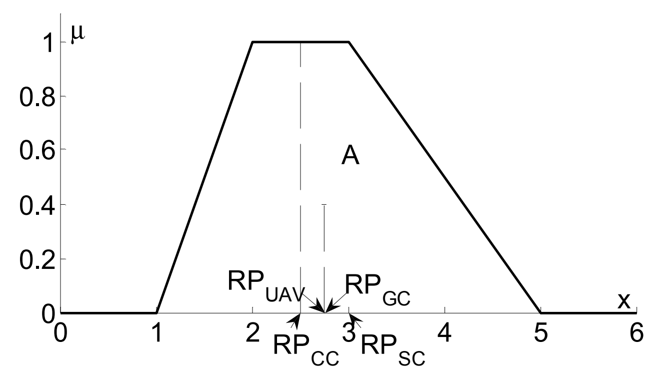

Figure 4. Reference point types

set. Further on we will use the centre of the core (RP_{CC}) as reference point of the fuzzy sets. Its value is calculated by

$$RP_{CC}(A) = \left(\inf\left\{[A]^1\right\} + \sup\left\{[A]^1\right\} \right)/2 \tag{3}$$

where $[A]^1$ is the α-cut of the fuzzy set A at the level of α=1.

The reference point of a relation $\widetilde{R}$ is that point of the multidimensional universe of discourse which is defined by the reference points of the component fuzzy sets

$$RP(\widetilde{R}) = \left\{ RP_1(\widetilde{R}), RP_2(\widetilde{R}), \ldots, RP_n(\widetilde{R}) \right\}. \tag{4}$$

B. Similarity Measurement using Fuzzy Subsethood Values

In interpolative fuzzy inference the similarity between the current fuzzy input and the antecedents of the known rules plays a determinant role in course of the calculation of the conclusion. Suppose the rules being of form

$$IF\ x_1 = A_{11}\ AND \ldots AND\ x_N = A_{N1}\ THEN\ y = B_1 \tag{5}$$

where x_i is the linguistic variable in the ith $i\in[1,n]$ antecedent dimension, A_{i1} ($A_{i1}\in X_i$) is a linguistic value in the same dimension, X_i is the universe of discourse of the ith $i\in[1,n]$ antecedent dimension, and y is the linguistic variable in the consequent dimension. The antecedent parts of the rules can be viewed as multidimensional fuzzy sets or n-ary fuzzy relations of form

$$\begin{aligned} \widetilde{R} = \{ & (x_1, x_2, \ldots, x_n), \mu_{\widetilde{R}}(x_1, x_2, \ldots, x_n) \mid \\ & (x_1, x_2, \ldots, x_n) \in X_1 \times X_2 \times \ldots \times X_n, \\ & \mu_{\widetilde{R}}(x_1, x_2, \ldots, x_n) \in [0,1] \}. \end{aligned} \tag{6}$$

The similarity between the n-dimensional observation represented by the n-ary relation $\widetilde{O}$ and the rule antecedent represented by the n-ary relation $\widetilde{R}$ depends on two components, i.e. the shape similarity and the distance between the reference points of the relations.

The shape similarity measures the closeness of the shapes of two fuzzy sets that share the same reference point. There are several solutions in the literature for this task. They can be grouped in four main categories:

- α-cut based techniques that are based on the differences between the endpoints of the sets' α-cuts. Most of the FRI methods (e.g. [3][6][7][9][10][21]) use this approach. Its drawback is its demand on a high number of α-cuts whose calculation is not always simple.
- Polar-cut based techniques that apply a polar coordinate system in the reference point of the sets and measure polar distances. For example FRIPOC [8] uses this approach. Its drawback is its demand on a high number of polar-cuts whose calculation sometimes is quite complex.
- Fixed point based techniques that compare the membership values at the same point of the universe of discourse. For example some of the FRI techniques proposed in [1] follow this way. It also requires several calculations; however, they are usually simpler than the ones in the previous cases.
- The vague environment based techniques. For example FIVE [12] uses this approach. Its drawback is that an exact form for the required scaling function can be calculated only in case of some membership function types. In other cases an approximated scaling function has to be determined.

In this paper, we follow the third approach using a similarity measure based on the fuzzy subsethood values [17]. The fuzzy subsethood value represents the degree to which a fuzzy set is a subset of another fuzzy set. For example in case of two discrete fuzzy sets A and B belonging to the universe X the fuzzy subsethood value of A to B is

$$FSV(A,B) = \frac{\sum_{x\in X} \mu_{A\cap B}(x)}{\sum_{x\in X} \mu_B(x)}, \tag{7}$$

where ∩ is an arbitrary t-norm. The value of $FSV(.)\in[0,1]$, the highest possible value corresponds to the case of two identical fuzzy sets. In order to apply FSV to the measurement of the similarity between fuzzy relations having an identical reference point we extend it to the multidimensional case. Thus the fuzzy subsethood value of the n-ary relation $\widetilde{O}$ to the n-ary relation $\widetilde{R}$ will be

$$FSV(\widetilde{O},\widetilde{R}) = \frac{\sum_{x_1\in X_1}\sum_{x_2\in X_2}\cdots\sum_{x_n\in X_n} \mu_{\widetilde{O}\cap\widetilde{R}}(x_1,x_2,\ldots,x_n)}{\sum_{x_1\in X_1}\sum_{x_2\in X_2}\cdots\sum_{x_n\in X_n} \mu_{\widetilde{R}}(x_1,x_2,\ldots,x_n)}, \tag{8}$$

where ∩ is the intersection between two relations, which is calculated by

$$\begin{aligned} & \mu_{\widetilde{O}\cap\widetilde{R}}(x_1,x_2,\ldots,x_n) = t\left[\mu_{\widetilde{O}}(x_1,x_2,\ldots,x_n), \mu_{\widetilde{R}}(x_1,x_2,\ldots,x_n)\right], \\ & \forall (x_1,x_2,\ldots,x_n) \in X_1\times X_2\times\ldots\times X_n, \end{aligned} \tag{9}$$

where t is an arbitrary t-norm.

In order to keep low the computational complexity of the similarity measurement we introduce a simplified measure of the fuzzy subsethood value ($SFSV$) of two fuzzy relations. SFSV works with the projections of the relations in each dimension

$$SFSV(\widetilde{O},\widetilde{R}) = \frac{\sum_{i=1}^{n} FSV\left(P_i(\widetilde{O}), P_i(\widetilde{R})\right)}{n}, \tag{10}$$

where $P_i(.)$ is the projection of the relation to the dimension i, i.e. the component fuzzy set of the relation in ith dimension. The value of $SFSV(.)\in[0,1]$, the highest possible value corresponds to the case of two identical fuzzy relations.

The second component of the proposed similarity measure aiming the comparison of the observation and the rule antecedents is their relative distance. One calculates first the

Euclidean distance between the reference points of the corresponding fuzzy relations by

$$d(\tilde{O},\tilde{R}) = d(RP(\tilde{O}), RP(\tilde{R})) = \sqrt{\sum_{i=1}^{n}(RP_i(\tilde{O}), RP_i(\tilde{R}))^2}, \qquad (11)$$

where $RP_i(.)$ is the reference point of the relation in the ith dimension. Next, we determine the relative distance by dividing $d(.)$ with the maximum possible distance, i.e. the distance between the point defined by the lower endpoints of the input ranges and the point defined by the upper endpoints of the input ranges

$$d_{\max} = \sqrt{\sum_{i=1}^{n}(x_{i\max} - x_{i\min})^2}, \qquad (12)$$
$$X_i = [x_{i\min}, x_{i\max}].$$

Thus the relative distance will be

$$d_{rel}(\tilde{O},\tilde{R}) = \frac{d(\tilde{O},\tilde{R})}{d_{\max}} \qquad (13)$$

The value of $d_{rel}(.) \in [0,1]$, the highest possible value corresponds to the maximum possible distance. Using the above mentioned two components the similarity of the relation $\tilde{O}$ describing the observation to the relation $\tilde{R}$ describing a rule antecedent will be

$$S(\tilde{O},\tilde{R}) = SFSV(\tilde{O},\tilde{R}) \cdot 0.5 + (1 - d_{rel}(\tilde{O},\tilde{R})) \cdot 0.5 \qquad (14)$$

supposing that both components have the same importance. Likewise, we can define the dissimilarity of two fuzzy relations as

$$D(\tilde{O},\tilde{R}) = 1 - S(\tilde{O},\tilde{R}). \qquad (15)$$

C. Fuzzy Rule Interpolation based on Subsethood Values

The Fuzzy Rule Interpolation based on Subsethood Values (FRISUV) aims the determination of the conclusion knowing the observation and the rules of the rule base. The key idea of the proposed method is that the reference point of the conclusion should be calculated from the position of the known rules' conclusion sets using the similarity between the current observation and the rule antecedents. The functional relationship is an extension of the well known Shepard crisp interpolation [18] which interpolates by calculating the weighted average of the known points' ordinates. Using this concept the reference point of the interpolated conclusion B^* will be

$$RP(B^*) = \begin{cases} \dfrac{\sum_{j=1}^{N_R} D(\tilde{O},\tilde{R})^{-1} \cdot RP(B_j)}{\sum_{j=1}^{N_R} D(\tilde{O},\tilde{R})^{-1}} & \text{if } D(\tilde{O},\tilde{R}) > 0 \\ RP(B_j) & \text{otherwise,} \end{cases} \qquad (16)$$

where B_j is the consequent fuzzy set of the jth rule.

Supposing that the shapes of the consequent partitions' linguistic terms are identical and the sets differ only in their position the shape of the interpolated conclusion also has to adhere to this regularity. Thus its form will be identical with the shapes of the known sets.

IV. NUMERICAL EXAMPLE

In this section, we use a numerical example to illustrate the application of FRISUV. We apply trapezoidal shaped fuzzy sets in case of each partition as well as in case of the observation. We use a vector based representation of the membership functions; each break-point of the shape being described by its abscissa and its ordinate value. Using normal trapezoidal membership functions further on we suppose the ordinates of each set's break-points being described by the vector

$$\mu = [0 \quad 1 \quad 1 \quad 0].$$

All partitions are normalized; the system is a MISO one formed of two input and one output dimensions. Let the fuzzy sets of the first input dimension be

$$A_1 = \begin{bmatrix} 0.05 & 0.15 & 0.20 & 0.30 \\ 0.20 & 0.30 & 0.35 & 0.45 \\ 0.70 & 0.80 & 0.85 & 0.95 \end{bmatrix},$$

where each row describes a fuzzy set. Similarly let the fuzzy sets of the second dimension be

$$A_2 = \begin{bmatrix} 0.05 & 0.15 & 0.20 & 0.30 \\ 0.35 & 0.45 & 0.50 & 0.60 \\ 0.70 & 0.80 & 0.85 & 0.95 \end{bmatrix}.$$

Suppose the sets of the consequent dimension are

$$B = \begin{bmatrix} 0.05 & 0.15 & 0.20 & 0.30 \\ 0.20 & 0.30 & 0.35 & 0.45 \\ 0.75 & 0.85 & 0.90 & 1.00 \end{bmatrix}.$$

The example system contains three rules, which are described by their antecedents ($\tilde{R}$) and consequents (C) as follows

$$\tilde{R} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 2 & 3 \end{bmatrix},$$

$$C = [1 \quad 2 \quad 3]^T$$

where each row represents a rule. Let the observation ($\tilde{O}$) be defined by the vectors

$$A_1^* = [0.50 \quad 0.55 \quad 0.60 \quad 0.65],$$

$$A_2^* = [0.60 \quad 0.65 \quad 0.70 \quad 0.75].$$

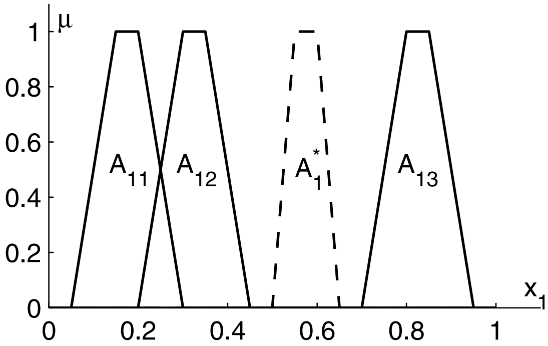

Figure 5. First antecedent dimension of the system and the observation set

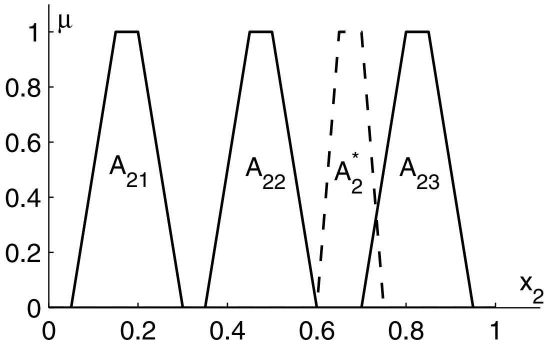

Figure 6. Second antecedent dimension of the system and the observation set

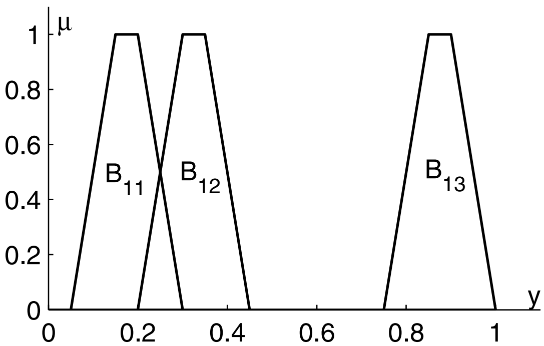

Figure 7. Consequent dimension of the system

Fig. 5, 6, and 7 represent the antecedent and consequent sets as well as the observation sets, which are drawn by dashed lines. Fig. 8 shows the rule antecedents and the observation as pyramid shaped relations in the antecedent space.

Next, we determine the reference points of the fuzzy sets. We use centre of the core type reference points that define the abscissa of the *RP*s as the arithmetical mean of the endpoints of the core. Thus the reference points rounded to two decimals are

$$RP(A_1)=[0.18 \quad 0.33 \quad 0.83]^T$$

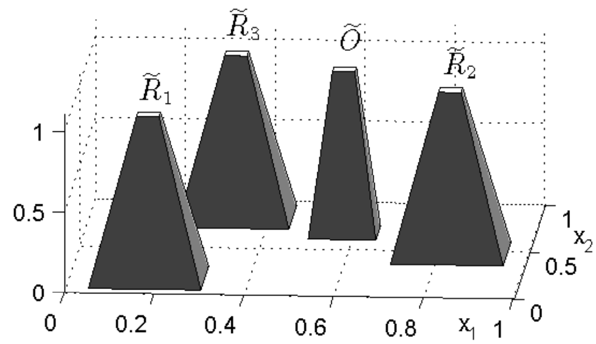

Figure 8. Rule antecedents and the observation as pyramid shaped relations in the antecedent space

$$RP(A_2)=[0.18 \quad 0.48 \quad 0.83]^T$$

$$RP(B)=[0.18 \quad 0.33 \quad 0.88]^T$$

$$RP(A_1^*)=0.58$$

$$RP(A_2^*)=0.68$$

Owing to the application of unit intervals in each antecedent dimension the maximum distance between the observation and rule antecedents is

$$d_{max}=\sqrt{2}=1.41$$

The distances between the observation sets and the antecedent sets in the first dimension are

$$D_1=RP(A_1^*)\cdot[1]_{3,1}-RP(A_1)=[0.58 \quad 0.58 \quad 0.58]^T-[0.18 \quad 0.33 \quad 0.83]^T$$

$$D_1=[0.40 \quad 0.25 \quad -0.25]^T$$

$$D_2=RP(A_2^*)\cdot[1]_{3,1}-RP(A_2)=[0.68 \quad 0.68 \quad 0.68]^T-[0.18 \quad 0.48 \quad 0.83]^T$$

$$D_2=[0.50 \quad 0.20 \quad -0.15]^T$$

The distances between the observation and the rule antecedent relations are

$$[d_{rel\,j}]=\left[\left(\sqrt{D_1(\widetilde{R}(j,1))^2+D_2(\widetilde{R}(j,2))^2}\right)/d_{max}\right]$$

$$D_{rel}=\begin{bmatrix}\left(\sqrt{D_1(1)^2+D_2(1)^2}\right)/1.41\\ \left(\sqrt{D_1(3)^2+D_2(2)^2}\right)/1.41\\ \left(\sqrt{D_1(2)^2+D_2(3)^2}\right)/1.41\end{bmatrix}=\begin{bmatrix}0.45\\0.23\\0.21\end{bmatrix}$$

In order to calculate the fuzzy subsethood values in each antecedent dimension we translate the observation into the position of each antecedent set

$$A_{T1}^*=[1]_{3,1}\cdot A_1^*-D_1\cdot[1]_{1,4}$$

$$A_{T1}^{*}=\begin{bmatrix}0.10 & 0.15 & 0.20 & 0.25\\ 0.25 & 0.30 & 0.35 & 0.40\\ 0.75 & 0.80 & 0.85 & 0.90\end{bmatrix}$$

$$A_{T2}^{*}=[1]_{3,1}\cdot A_{2}^{*}-D_{2}\cdot[1]_{1,4}$$

$$A_{T2}^{*}=\begin{bmatrix}0.10 & 0.15 & 0.20 & 0.25\\ 0.40 & 0.45 & 0.50 & 0.55\\ 0.75 & 0.80 & 0.85 & 0.90\end{bmatrix}$$

We define 101 uniformly distributed points on the unit interval for the calculation of the *FSV*s. For space saving purposes here only the results of the *FSV* calculation are shown. The fuzzy subsethood values of the observation to the antecedent sets in the first dimension are

$$FSV_1=[0.67 \quad 0.67 \quad 0.67]^T.$$

Similarly the fuzzy subsethood values of the observation to the antecedent sets in the second dimension are

$$FSV_2=[0.67 \quad 0.67 \quad 0.67]^T.$$

The simplified fuzzy subsethood values of the observation to the antecedent relations are

$$SFSV=\begin{bmatrix}(FSV_1(1)+FSV_1(1))/2\\ (FSV_1(3)+FSV_1(2))/2\\ (FSV_1(2)+FSV_1(3))/2\end{bmatrix}=\begin{bmatrix}0.67\\ 0.67\\ 0.67\end{bmatrix}$$

The similarity of the observation and the rules antecedents is expressed by

$$S=\left(SFSV+[1]_{3,1}-D_{rel}\right)/2=\left([0.67 \quad 0.67 \quad 0.67]^T+[1 \quad 1 \quad 1]^T-[0.45 \quad 0.23 \quad 0.21]^T\right)/2$$

$$S=[0.61 \quad 0.72 \quad 0.73]^T.$$

The dissimilarity of the observation and the rules antecedents is characterized by the matrix

$$D=[1]_{3,1}-S=[0.39 \quad 0.28 \quad 0.27]^T.$$

The reciprocal values of the dissimilarity measures are

$$D_R=[1]_{3,1}./D=[2.54 \quad 3.57 \quad 3.71]^T,$$

where ./ denotes the element-by-element division of two matrices of the same size. The denominator of the formula (16) is

$$Den=2.54+3.57+3.71=9.82.$$

The reference point of the conclusion is

$$RP(B^*)=(2.54\cdot 0.18+3.57\cdot 0.33+3.71\cdot 0.88)/9.82=0.49.$$

We use the first set of the consequent dimension as prototype and we calculate the abscissas of the conclusion's break-points by translating this set into the position of the conclusion. The distance between the first set and the conclusion is

$$d_y=RP(B_1)-RP(B^*)=0.18-0.49=-0.31.$$

Thus the break-points of the conclusion are

$$B^*=B_1-d_y\cdot[1]_{1,4}=[0.05 \quad 0.15 \quad 0.20 \quad 0.30]+0.31\cdot[1 \quad 1 \quad 1 \quad 1],$$

$$B^*=[0.36 \quad 0.46 \quad 0.51 \quad 0.61].$$

Fig. 9 shows the conclusion in the consequent dimension using dashed line.

V. CONCLUSIONS

The presented method is applicable in case of any valid set shape type on the antecedent and consequent side. The only restriction is that the linguistic terms of the consequent partition should have identical shapes. FRISUV results always a valid fuzzy set as conclusion and is applicable independent of the number of antecedent dimensions. The method ensures compatibility with the rule base, namely if an observation is identical with the antecedent part of a rule the conclusion produced by FRISUV corresponds to the consequent part of that rule.

Most of the required calculations easily can be vectorized, which speeds up essentially the interpolation process in case of a Matlab implementation. FRISUV has low computational complexity, which results in a quick determination of the conclusion. The presented method also can be used in fuzzy controllers applying for example the development methods introduced by Precup Doboli and Preitl [15]; or Precup et al. [16]; or Botzheim, Hámori and Kóczy [2], or Škrjanc, Blažič and Agamennoni [19].

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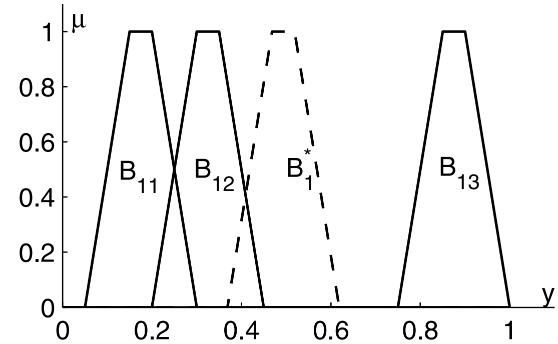

Figure 9. First antecedent dimension of the system and the observation set

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