

Fuzzy Model based Prediction of Ground-Level Ozone Concentration

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Abstract: Ground-level ozone is a dangerous pollutant for which the prediction of the concentration could be of great importance. In this paper, we present and compare three fuzzy models aiming the forecasting of ground-level ozone concentration. The models apply Takagi-Sugeno, respective LESFRI fuzzy inference techniques and were generated using the ANFIS method of the Matlab's Fuzzy Logic ToolBox, respective the RBE-DSS method of the SFMI toolbox. Although all of the methods proved to be applicable the model using LESFRI ensured the best results with a low number of rules.

Keywords: *Ground-level ozone, fuzzy models, LESFRI, ANFIS, RBE-DSS*

1. Introduction

The analysis and forecasting of air quality parameters are important topics of atmospheric and environmental research. In many of the applications, data are generated in the form of a time series. Therefore, time series analysis is a major task in forecasting average ozone concentrations, where one tests and predicts known or estimated observations for past times using them as input into the model to see how well the output matches the known observations.

Ground-level ozone (O_3) is one of the air pollutants of most concern in Europe. It is an irritating and reactive component in atmosphere that has negative impacts on human health, climate, vegetation and materials [23].

Ground-level ozone is a highly reactive oxidant and is unique among pollutants because it is not emitted directly into the air [20]. It is a secondary pollutant that results from complex chemical reactions in the atmosphere. In the presence of the sun's ultraviolet radiation (RAD), oxygen (O_2), nitrogen dioxide (NO_2), and volatile organic compounds (VOCs) react in the atmosphere to form ozone and nitric oxide (NO) through the reactions given in (1) and (2)



With regards to the prediction of O_3 concentrations, several studies have been published. Sousa, Martins, Alvim-Ferraz, and Pereira [28] applied multiple linear regression (MLR) and artificial neural networks (ANNs); Ozdemir, Demir, Altay, Albayrak, and Bayat [21] used ANNs; Al-Alawi, Abdul-Wahab, Bakheit [1] combined principal component regression and ANNs; Pires, Martins, Pereira and Alvim-Ferraz [22] developed three different models an MLR based, an ANN based and one based on multi-gene genetic programming (MGP), from which the last one (MGP) ensured the best predictions.

Fuzzy systems have been used successfully for numerous practical applications. Kovács and Kóczy [18] developed a fuzzy rule interpolation (FRI) based model for behaviour-based control structures; Johanyák and Ádámné [9] constructed fuzzy models for the prediction of thermoplastic composites' mechanical properties; Wong and Gedeon [34] as well as Johanyák and Kovács [13] developed FRI based systems for prediction of petrophysical properties. Hládek, Vaščák and Sinčák [5] proposed a hierarchical multi agent control system based on rule based fuzzy system for pursuit-evasion task. Despite their advantages and wide applicability area fuzzy logic based solutions for ozone concentration prediction have not been published previously.

Therefore our research aimed the development and analysis of two types of fuzzy systems one applying a traditional Takagi-Sugeno [29] inference method using a dense rule base and another applying fuzzy rule interpolation (FRI) based reasoning technique using a sparse rule base. The results proved the applicability of the above mentioned methods in this case as well.

The rest of this paper is organized as follows. Section II reviews briefly the applied methods. Section III introduces the experiments the data came from and the results of the modelling. The conclusions are drawn in section IV.

2. Fuzzy Modeling and Inference

A fuzzy rule based system describes usually a nonlinear mapping between inputs and outputs based on fuzzy set concept. One can assign to set A a characteristic function $x_A: X \rightarrow \{0,1\}$, which can take only the 0 or 1 (crisp) numerical values in case of the classical set concept (3) and values from a continuous interval (usually $[0,1]$) in case of the fuzzy concept [35].

$$x_A = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

In fuzzy logic the mapping of crisp inputs x^* to crisp outputs y^* generally is solved in three steps, which can be seen on Figure 1. where $x^*=(x_1^*, x_2^*, \dots, x_n^*)$ is the input, $y^*=(y_1^*, y_2^*, \dots, y_m^*)$ is the output, n is the number of input dimensions, and m is the number of output dimensions.

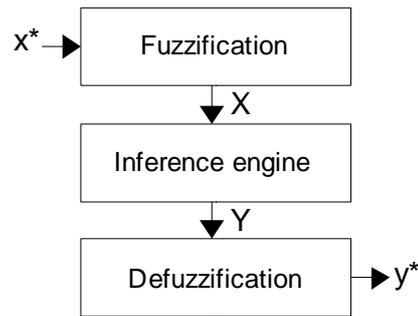


Figure 1. Block diagram of functioning of a fuzzy rule based system

Depending on the number of input and output linguistic variables (dimensions) one can define four groups of fuzzy systems, i.e. multiple-input multiple-output (MIMO), multiple-input single-output (MISO), single-input multiple-output (SIMO), single-input single-output (SISO). In the case of ozone concentrations' forecasting we used MISO models. In the following subsections we review shortly the methods and tools we used for the generation of the three fuzzy models aiming the better prediction of ozone concentration.

2.1. Takagi-Sugeno type fuzzy inference

The mapping of inputs to outputs in a fuzzy system is determined by a set of “IF–THEN” rules of form

$$\text{If } X \text{ is } A^i \text{ then } Y \text{ is } B^i ; i = 1, \dots, R, \quad (4)$$

where in case of a MISO system $X = (x_1, x_2, \dots, x_n)$ consists of a set of input variables, Y is the output variable, and R is the number of rules [32]. The fuzzy sets $A^i = (A_1^i, A_2^i, \dots, A_n^i)$ and B^i are the antecedent and consequent parts of the fuzzy rules.

The Takagi–Sugeno type fuzzy system [29] also called “functional fuzzy system”, uses a function g^i instead of a linguistic term

$$\text{If } X \text{ is } A^i \text{ then } y^i \text{ is } g^i ; i = 1, \dots, R, \quad (5)$$

where the consequents $g^i = f(X)$. When the values g^i are constants the system is called zero order Takagi-Sugeno system. The crisp output of the fuzzy system is determined by

$$y = \frac{\sum_{i=1}^R w_i g_i}{\sum_{i=1}^R w_i}. \quad (6)$$

where w_i is the firing strength of the i th rule. Li Xin Wang [33] proved that any continuous function can be approximated by zero order Takagi-Sugeno systems.

2.2. ANFIS, Adaptive-Network-Fuzzy Inference System

The Matlab's ANFIS software generates a Takagi-Sugeno type fuzzy system from sample data using an adaptive neural network [6]. An adaptive network can be considered in some sense as the generalization of neural networks and fuzzy systems [6][7]. The typical structure of an adaptive network is shown in Figure 2. The network consists of nodes connected by directed edges. The typical adaptive network does not contain any feedback and it is organized in layers. The inputs and outputs of the adaptive network are denoted by x_i and O_i^L . The number of layers is L . The number of nodes in the k -th layer is denoted by $\#(k)$. Figure 3. shows a simple example of an adaptive network.

2.3. LESFRI

In many cases the dense rule base (e.g. Figure 4.) demanded by the classical compositional fuzzy inference techniques contains a large number of rules that increases exponentially with the number of input dimensions which fact also increases the computational complexity and the storage demand.

This problem led to the development of fuzzy systems that are able to produce the output relying only on a minimal set of rules. Thus it is not necessary to ensure a full coverage of the antecedent space by rules and a sparse rule base with low complexity can be applied (see Figure **Hiba! A hivatkozási forrás nem található.**).

The development of Fuzzy Rule Interpolation (FRI) based Inference Techniques (FRITs) gives new methodology on the field for practical applications due to the reduced complexity and storage space demand as well as due to its ability to handle cases when there are no rules that would describe the expected output for all the possible inputs.

FRITs can be divided into two groups depending on whether they are producing the estimated conclusion directly or they are interpolating an intermediate rule first.

Relevant members of the first group are among others the linear rule interpolation (KH method) [15] proposed by Kóczy and Hirota, which is the first developed one, the MACI (Tikk and Baranyi) [30], the FIVE [17] introduced by Kovács and Kóczy as well as the interpolation method developed by Kovács [16] that extended the fuzzy interpolation to the general metric spaces.

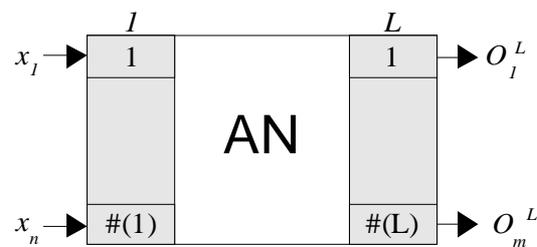


Figure 2. The layer structure of an adaptive network [19]

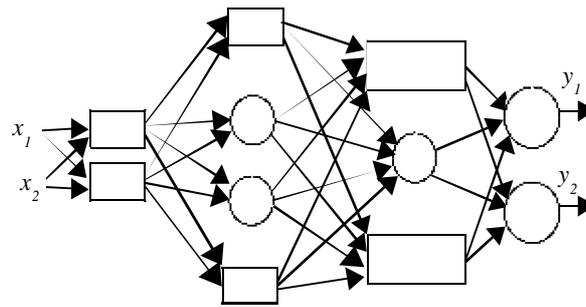


Figure 3. Simple example for an adaptive network [19]

The methods belonging to the second group follow the concepts laid down by the generalized methodology (GM) defined by Baranyi et al. in [2]. Typical members of this group are e.g. the technique family proposed by Baranyi et al. in [2], the ST method suggested by Yan, Mizumoto and Qiao [31], the transformation based technique published by Chen and Ko [4] as well as the techniques LESFRI [11], FRIPOC [12] and VEIN [14] developed by Johanyák and Kovács.

We chose LESFRI (LEast Squares based Fuzzy Rule Interpolation) [11] for the task of FRI based fuzzy inference. It was applied owing to the good practical experiences (e.g. [9]) in course of previous applications. In its first step LESFRI interpolates a new rule into the position of the observation. The task is solved in three phases. Firstly, the antecedent membership functions are calculated using the FEAT-LS (Fuzzy sEt interpolATIOn based on method of Least Squares) fuzzy set interpolation method. Next, one determines the position (reference points) of the consequent linguistic terms of the new rule using an adapted version of the Shepard interpolation [26]. Thirdly, the shapes of the consequent sets are calculated using the same set interpolation technique (FEAT-LS) as in the first phase.

LESFRI determines the conclusion in its second step using the single rule reasoning method SURE-LS (Single rUle REasoning based on the method of Least Squares) that

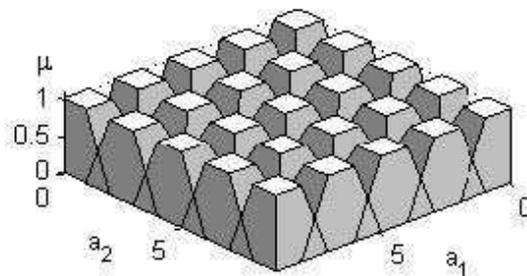


Figure 4. Antecedent space of a dense rule base

calculates the necessary modifications of the new rule's consequent sets based on the dissimilarities between the rule antecedent and observation sets.

2.3.1. FEAT-LS

The FEAT-LS method aims the determination of a new linguistic term in a fuzzy partition based on a supposed regularity between the known sets of the partition. First all linguistic terms are shifted horizontally into the interpolation point and next, one calculates the shape of the new set from the overlapped membership functions (A^i Figure 5 right side).

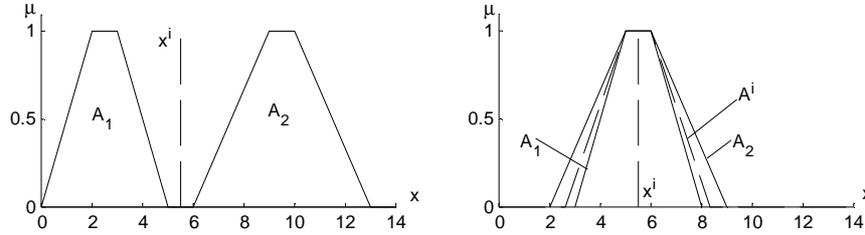


Figure 5. Original partition and interpolation point (x^i)

FEAT-LS targets the preservation of the characteristic shape type of the partition (e.g. trapezoidal on Figure 5) therefore it applies the method of the weighted least squares for the identification of the new set's parameters. The weighting is related to the original distance between the sets and the interpolation point. The calculations are done α -cut wise using only the α -levels corresponding to the characteristic points of the partition's default shape type.

2.3.2. SURE-LS

The revision method SURE-LS (Single rUle REasoning based on the method of Least Squares) is a special fuzzy inference technique that takes into consideration only one rule for the determination of the conclusion. The method is applicable when its antecedent sets are in the same position as the observation sets in each antecedent dimension and the heights (maximal membership value) of all involved fuzzy sets are the same.

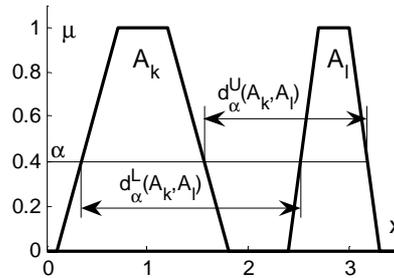


Figure 6. Lower ($d_\alpha^L(A_k, A_l)$) and upper ($d_\alpha^U(A_k, A_l)$) fuzzy distance at the α -level

SURE-LS calculates the conclusion by modifying the consequent sets of the rule. This modification is related to the similarity between the antecedent linguistic terms and the observation sets, which is measured independently in each input dimension by the means of their fuzzy distance (see Figure 6) and is aggregated by calculating the average distance.

2.4. RBE-DSS

In course of the rule base generation one can follow two different approaches. The first one divides the task in two separate steps, i.e. the structure definition and the parameter identification (e.g. Precup, Doboli and Preitl [24]; or Botzheim, Hámori and Kóczy [3], or Škrjanc, Blažič and Agamennoni [27]).

The second approach works incrementally by simultaneously modifying the structure and the parameters, i.e. introducing or eventually eliminating rules and tuning the parameters of the membership functions (e.g. Johanyák and Kovács [10]).

The Rule Base Extension with Default Set Shapes (RBE-DSS) [10] starts with an empty rule base and a set of training data points given in form of coherent input and output values. First the starting rule base is defined by determining the first two rules. They aim the description of the minimum and maximum output. One seeks the two extreme output values and a representative data point for each of them. If several data points correspond to an extreme value, one should select the one that is closer to an endpoint of the input domain.

Next, a tuning algorithm starts aiming the identification of the parameters of the initial fuzzy sets. This algorithm uses an iterative approach adjusting each parameter in several steps separately. The system is evaluated in each iteration step for different parameter values against a training data set and the parameter values ensuring the best performance index are kept for the next iteration.

If the decreasing velocity of the performance index of the system is too slow, i.e. it falls below a specified threshold after two consecutive iterations a new rule is generated. It is because the system tuning reached a local or global minimum of the performance index and the performance cannot ameliorate further by the applied parameter identification algorithm. The new rule introduces additional tuning possibilities.

In order to create the new rule, one seeks for the calculated data point, which is the most differing one from its corresponding training point. The input and output values of this training point will be the reference points of the antecedent and consequent sets of the new rule.

3. Experimental data and modeling results

The air pollution data were collected in an urban site of Northern Portugal with traffic influences situated in Oporto [22]. The site is situated on the left edge of the Douro River, at an altitude of 90 m approximately. The study period was two weeks of July 2004, where high O₃ concentrations were measured and there was no missing data.

In course of the experiments 10 characteristics were measured: the hourly average concentrations (in $\mu\text{g}/\text{m}^3$) of carbon monoxide (CO), nitrogen oxides (NO, NO₂ and

NO_x) and O₃; hourly averages of air temperature (T), solar radiation (RAD), relative humidity (RH) and wind speed (WS); the day of week (DW; the O₃ behavior is different on weekdays and on weekend). All environmental and meteorological inputs corresponded to the same hour of the previous day.

Based on the results published in [22] we took into consideration in course of the modeling only the most important factors that are T, RH, O₃, NO₂, NO. We formed two groups of the experimental data: one containing 259 measurements for system training purposes and one with 84 measurements for testing purposes. The test data were selected randomly from the original sample.

The quality of a fuzzy model is measured using a performance index that aggregates the distances between the measured and calculated output points. One can choose from several possible performance indices available in the literature (e.g. in [25]). We used the root mean square of the error (RMSE) as performance index owing to its good comprehensibility and comparability to the range of the output linguistic variable.

3.1. Modeling results using ANFIS and Takagi-Sugeno inference

We created two fuzzy models using the ANFIS software. The first one (labeled as O3_Anfis_3S_Trimf_Corr.fis) was a zero ordered Takagi-Sugeno model having triangle shaped membership functions and three fuzzy sets in each dimension. We used the hybrid training algorithm with three epochs. Figure 7. and Figure 8. present the measured and calculated output points in case of the training respective test data sets.

The second fuzzy system (labeled as O3_Anfis_5S_Trimf_Corr.fis) was a zero ordered Takagi-Sugeno model having triangle shaped membership functions and five fuzzy sets in each dimension. We used the hybrid training algorithm with five epochs. Figure 9. and Figure 10. present the measured and calculated output points in case of the train respective test data sets.

The performance of the systems was measured using the root mean square of the error (RMSE). The numerical results are summarized in Table I. In case of both systems one can identify clearly a slightly overfitting of the models to the training data.

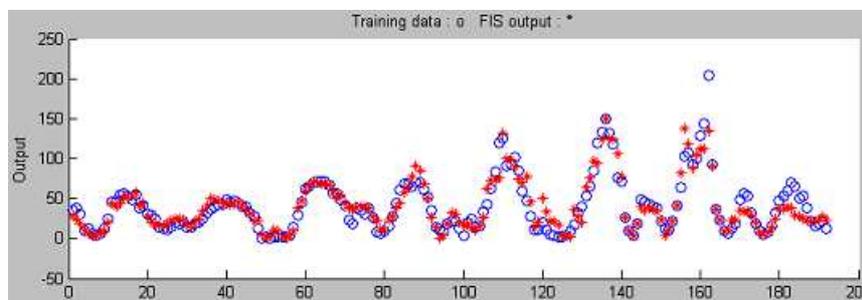


Figure 7. Measured and calculated output points in case of the first fuzzy system and the training data set

Table 1. System performance (RMSE) in case of the training and testing data

	Training	Test	Number of rules
O3_Anfis_3S_Trimf_Corr.fis	10.5101	95.0337	243
O3_Anfis_5S_Trimf_Corr.fis	4.4400	105.9679	3125
O3_2R_Reduced_01_640_00705.fis	8.0007	14.8703	66

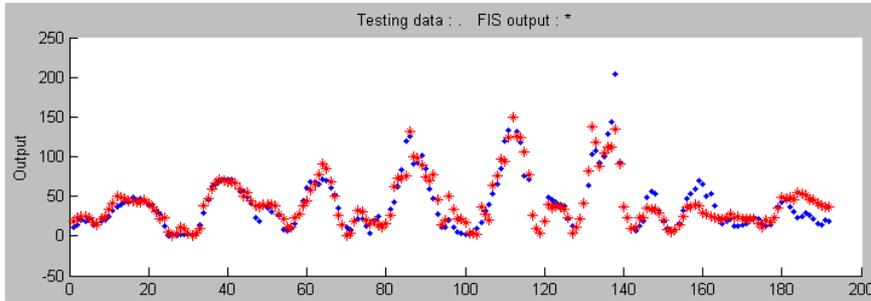


Figure 8. Measured and calculated output points in case of the first fuzzy system and the testing data set

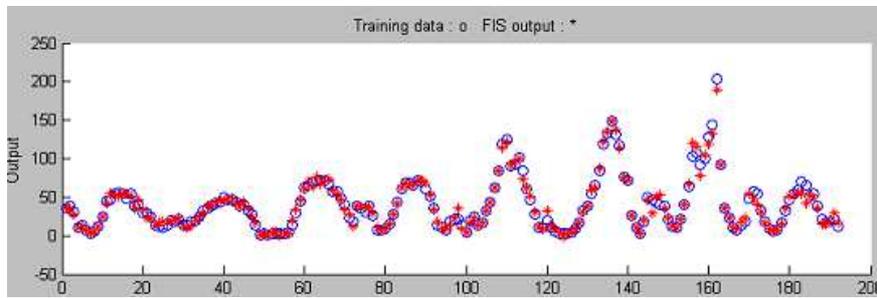


Figure 9. Measured and calculated output points in case of the second fuzzy system and the training data set

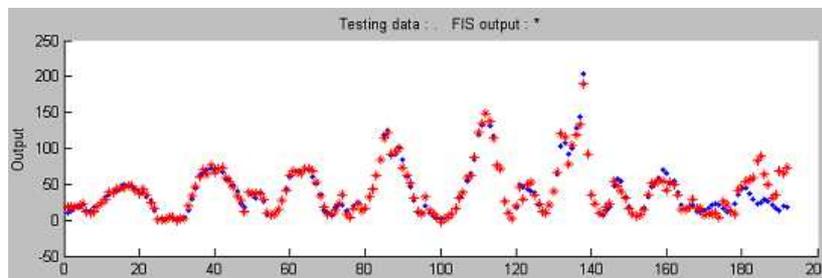


Figure 10. Measured and calculated output points in case of the second fuzzy system and the testing data set

3.2. Modeling results using RBE-DSS and LESFRI

We also created a fuzzy model using the SFMI toolbox [8]. The selected model identification method was RBE-DSS and we used LESFRI for fuzzy inference in the resulting sparse rule base. The system performance (RMSE) in case of the training data set was between the results obtained in case of the two ANFIS created systems (see Table I). On the other hand, there was a much smaller overfitting, i.e. this system presented the best performance in case of the test data. Besides, the number of rules necessary for the description of the relation between the input and output variables was the smallest in the case of the third fuzzy system. Figure 11. and Figure 12. present the measured and calculated output points in case of the train respective test data sets. The numerical results are summarized in Table I.

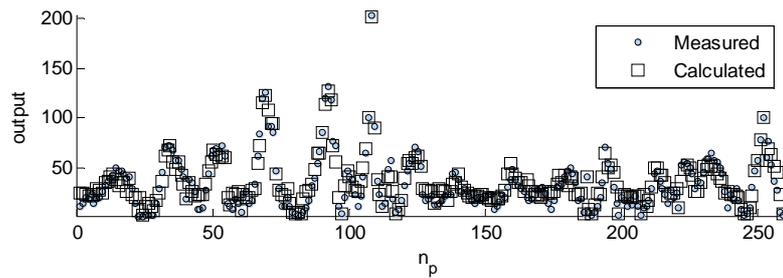


Figure 11. Measured and calculated output points in case of the third fuzzy system and the training data set

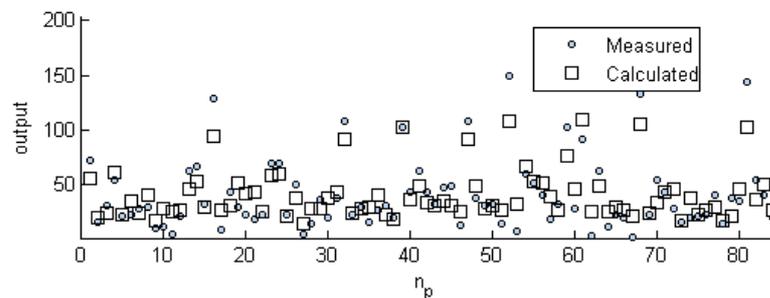


Figure 12. Measured and calculated output points in case of the third fuzzy system and the testing data set

4. Conclusion

This paper presented the application of two different fuzzy rule base generation approaches in order to model the relation between five environmental characteristics

and ground level ozone concentration. The aim of our research was the creation of fuzzy models that can be used in practice for the prediction of the ozone level.

In our case the fuzzy system applying a sparse rule base and inference based on fuzzy rule interpolation ensured the best results taking into consideration both the training and testing data samples. This solution ensured slightly better performance than the previously applied approaches published e.g. in [22].

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